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Relative performance of AUSGeoid09 in mountainous terrain

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Abstract: The Australian Height Datum (AHD) is the national vertical datum for Australia, and AUSGeoid09 is the latest quasigeoid model used to compute (normal-orthometric) AHD heights from Global Navigation Satellite System (GNSS) derived ellipsoidal heights. While previous studies have evaluated the AUSGeoid09 model across Australia, such studies have generally not focused on mountainous terrain. This paper investigates the performance of AUSGeoid09 in a relative sense in the Mid Hunter and Snowy Mountains regions of New South Wales, from a user's perspective. Relative (i.e. height difference between two points) comparisons were undertaken between AUSGeoid09-derived heights and official AHD heights. The performance of AUSGeoid09 was compared to its predecessor AUSGeoid98. In both study areas, an overall improvement was evident when applying AUSGeoid09 to compute AHD height differences. AUSGeoid09 generally provided AHD height differences at the ±0.05 m to ±0.09 m level (1 sigma) and substantially increased the percentage of GNSS-derived height differences meeting 3rd order differential levelling specifications. This is a very encouraging result, considering the difficulties of spirit levelling in mountainous terrain and the increasing popularity of GNSS-based height transfer in practice.

Keywords: AUSGeoid09, Australian Height Datum, geoid model, Global Navigation Satellite System, N values, national datums

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1 Introduction

The Australian Height Datum (AHD) is the first and only national vertical datum for Australia. It was defined by setting to zero the average Mean Sea Level (MSL) values of 32 tide gauges around Australia for a period of about two years that began in 1966 and adjusting 97,230 km of 2-way spirit levelling [1]. Now, almost 50 years later, it is well known that shortcomings in the AHD realisation (AHD71 for mainland Australia and AHD83 for Tasmania) resulted in MSL not being coincident with the geoid at the tide gauges involved.

These shortcomings included not considering dynamic ocean effects (e.g. winds, currents, atmospheric pressure, temperature and salinity), a lack of long-term tide gauge data, and the omission of observed gravity. This has introduced considerable distortions of up to about 1.5 m into AHD across Australia, which is therefore considered a third-order datum [2, 3]. However, AHD continues to be a practical height datum that provides a sufficient approximation of the geoid for many surveying and engineering applications. Consequently, in surveying and engineering practice, AHD heights are often accepted as being equivalent to orthometric heights.

Over the last two decades, Global Navigation Satellite System (GNSS) technology has become the primary positioning tool due to its accuracy, speed and accessibility. GNSS-based heights refer to a reference ellipsoid, i.e. a purely mathematical representation of the earth, and therefore have no physical meaning. In most practice, however, heights are required that correctly reflect the flow of fluids, e.g. for drainage and pipeline design. Hence, a reliable geoid model is required to derive AHD heights from measured ellipsoidal heights.

N values (N), also known as geoid undulations or geoid-ellipsoid separations, can be used to convert GNSS-derived ellipsoidal heights (h) to AHD heights (H) and vice versa (provided N and h refer to the same ellipsoid):

$$H = h - N \tag{1}$$

This relationship ignores errors caused by the deflection of the vertical. However, these errors amount to less than a millimetre across Australia and can therefore be ignored [4].

For many years, the use of geoid models – or quasigeoid models, see e.g. [5, 6] for a discussion of the difference – has helped GNSS users to compute AHD heights from ellipsoidal heights. In the Australian context, AUSGeoid09 is the latest quasigeoid model that best fits AHD [7, 8].

While the performance of AUSGeoid09, along with the improvements it provides over its predecessor AUSGeoid98, has been investigated previously [7, 9], very few studies have focused on mountainous regions [10, 11]. Considering that gravity can change dramatically within a few kilometres on the earth's surface in Australia [12], especially in mountainous terrain, and that observed gravity data are generally sparse in these areas, it is necessary to evaluate the performance of AUSGeoid09 in mountainous regions in particular.

Geoid or quasigeoid models are commonly verified by using GNSS and orthometric height data. This can be done in an absolute and relative sense [13]: An absolute verification estimates the accuracy and precision of the (quasi)geoid, with respect to the geocentric ellipsoid, using GNSS networks that have been tied to an (inter)national reference frame and spirit-levelled orthometric heights that have been tied to the (national) vertical datum. A relative verification utilises GNSS-derived ellipsoidal height differences and spirit-levelled orthometric height differences to estimate the accuracy and precision of the (quasi)geoid gradients.

Previously the authors have evaluated AUSGeoid09 performance in the mountainous regions of the Mid Hunter and the Snowy Mountains in New South Wales (NSW) in an absolute sense [10]. It was found that AUSGeoid09 provides superior connection to AHD compared to its predecessor AUSGeoid98. However, a slope was detected for AUSGeoid09 residuals in the Snowy Mountains study area, illustrating that some discrepancies still remain between AUSGeoid09-derived heights and AHD.

This paper revisits these study areas and the previous datasets to investigate, from a user's point of view, AUSGeoid09 performance in a relative sense, using GNSS-derived ellipsoidal heights and official AHD heights on public record. A comparison between AUSGeoid09 and its predecessor AUSGeoid98 is again performed, but now in regards to height differences.

2 AUSGeoid09

AUSGeoid09 was released in March 2011 by Geoscience Australia to replace the previous model, AUSGeoid98 [14]. Both models refer to the GRS80 ellipsoid, which was adopted as the reference ellipsoid for the Geocentric Datum of Australia 1994 (GDA94), and cover the same geographical area between 108°E and 160°E longitude and between 8°S and 46°S latitude. However, AUSGeoid09 is provided as a 1' by 1' grid (approximately 1.8 by 1.8 km), making it four times denser than its predecessor [8].

Previous versions of AUSGeoid were predominantly gravimetric-only quasigeoids, and it was assumed that these were sufficiently close approximations of AHD – an assumption we now know to be incorrect. In contrast, AUSGeoid09 is a combined gravimetric-geometric quasigeoid, providing a direct connection to AHD and thereby allowing a more reliable determination of AHD heights from GNSS observations [7]. The empirically derived geometric component accounts for the offset between the gravimetric quasigeoid and AHD, which is predominantly caused by AHD not taking into account sea surface topography including the differential heating of the oceans.

Since the warmer or less dense water off northern Australia is about 1 metre higher than the cooler or denser water off southern Australia, AHD is about 0.5 m above the quasigeoid in northern Australia and roughly 0.5 m below the quasigeoid in southern Australia [7, 9]. The introduction of the geometric component takes care of most of this 1-metre trend across Australia (0.6-metre trend across NSW), thereby providing a better overall fit to AHD.

AUSGeoid09 has been shown to convert ellipsoidal heights to AHD heights with an accuracy of ±0.03 m (1 sigma) across most of Australia, with the exception of some pockets where the misfit can be larger than ±0.1 m due to errors caused by factors such as the ageing levelling network, geoid height variability or data deficiency [7]. Using a more practical approach, [9] found that AUSGeoid09 generally allows GNSS-based height determination in NSW at the ±0.05 m level (1 sigma). In contrast, its predecessor AUSGeoid98 only provides an absolute accuracy of ±0.4 m [14, 15].

Using Network Real Time Kinematic (NRTK) GNSS observations in the Blue Mountains area of NSW, [11] found that AUSGeoid09 allows AHD height determination at the ±0.03 m level (1 sigma) in flat terrain and at the ±0.06 m level (1 sigma) in mountainous terrain. However, results also indicate that there is room for improvement in regards to future versions of the AUSGeoid model for elevations above 500‘ m.

Similar AUSGeoid09 performance was reported by [10] who investigated two extensive datasets in the Mid Hunter and Snowy Mountains regions of NSW in an absolute sense. However, again some inconsistencies were detected between AUSGeoid09-derived heights and official AHD heights.

3 Absolute and relative GNSS heighting

In an absolute sense, also known as the single point approach, the GNSS-derived ellipsoidal height at a point can be converted to a (normal-orthometric) AHD height using Eq. (1). This is generally applied when users connect to a GNSS Continuously Operating Reference Station (CORS) network, e.g. via Network Real Time Kinematic (NRTK) observations, or use Precise Point Positioning (PPP) to derive AHD heights [16, 17].

In a relative sense, the AHD height of a point B is calculated relative to another point A with known AHD height, using the differences (Δ) in GNSS-derived ellipsoidal heights and N values supplied by the geoid model:

$$H_B = H_A + \Delta H_{AB} \text{ with } \Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \quad (2)$$

The advantage of the relative method is that simultaneous observations at both points minimise most of the systematic errors by virtue of the difference, as is the case with GNSS baseline processing [18, 19]. The absolute N values at both points may have relatively large errors but the height of point B is only contaminated by the small difference of these errors (ignoring any GNSS observational errors). Figure 1 illustrates these two approaches.

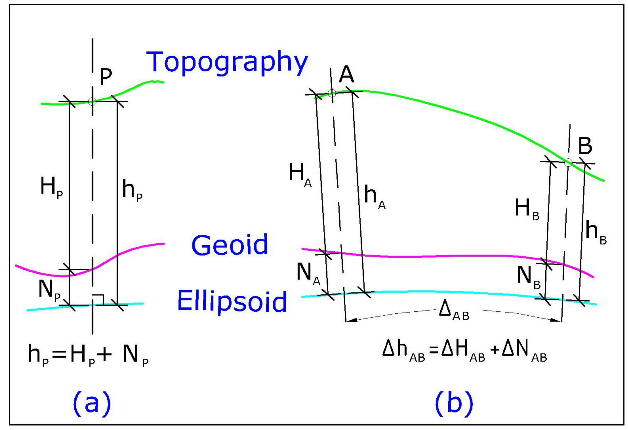

Figure 1: Relationship between ellipsoidal height (h), normal-orthometric AHD height (H) and geoid-ellipsoid separation (N) for (a) absolute and (b) relative GNSS heighting.

4 Relative geoid model verification

Following the recent absolute performance evaluation of AUSGeoid09 in two mountainous regions of NSW [10], this paper presents further investigation of AUSGeoid09 performance using the relative approach. While it is recognised that the growing popularity of CORS networks and PPP for GNSS-based height transfer relies on absolute AUSGeoid09 performance, from a practical point of view the relative verification may still be considered a more realistic approach than the absolute verification because it is based on the difference in height over a baseline. Surveyors generally continue to use this baseline method to 'carry' heights from established marks to unestablished marks. As indicated earlier, this concept is less affected by errors in the N values since common systematic errors are minimised by virtue of differencing.

Baselines were observed between points to obtain the differences in ellipsoidal heights, which were then converted into AHD height differences using the computed N values. This can be applied over all possible mark-to-mark vector combinations within a network of control points. However, a least squares adjustment is required to create a network with consistent ellipsoidal heights. The number of possible vector combinations is $n(n\text{-}1)/2$ where n denotes the number of points.

AUSGeoid09 and AUSGeoid98 performance was investigated by comparing between any two points the difference in GNSS-derived AHD heights (ΔH_{GNSS}) computed using Eq. (2) to the difference in official AHD heights (ΔH_{AHD}) for these two points. Residuals (R) were then computed as follows:

$$R = \Delta H_{AB(GNSS)} - \Delta H_{AB(AHD)} \quad (3)$$

The following four tests were performed, both statistically and graphically:

- Comparison over all observed baselines.
- Comparison over all possible mark-to-mark combinations.
- Comparison over all possible mark-to-mark combinations up to 100 km in length.
- Comparison over all possible mark-to-mark combinations as a function of AHD height difference.

Descriptive statistics were computed to obtain a numerical representation of the population sample, as previously adopted by [7]. Z-statistics were employed to identify any outliers, in this paper defined as three times larger than the standard deviation. Since it is necessary to consider residuals of different signs, the Root Mean Square (RMS) was also utilised.

The primary aim of this verification was to quantify the accuracy of AUSGeoid09 in regards to computing AHD height differences in mountainous terrain. A comparison between AUSGeoid09 and its predecessor AUSGeoid98 was performed to quantify the expected improvement of the former over the latter in mountainous areas. Furthermore, the baseline residuals were compared to the allowable misclose of 3rd order differential levelling [20], i.e. the maximum allowable misclose of $12\sqrt{d}$ (d = distance in km, here calculated on the GRS80 ellipsoid using Vincenty's inverse formula). Residuals were also expressed as parts per million (ppm).

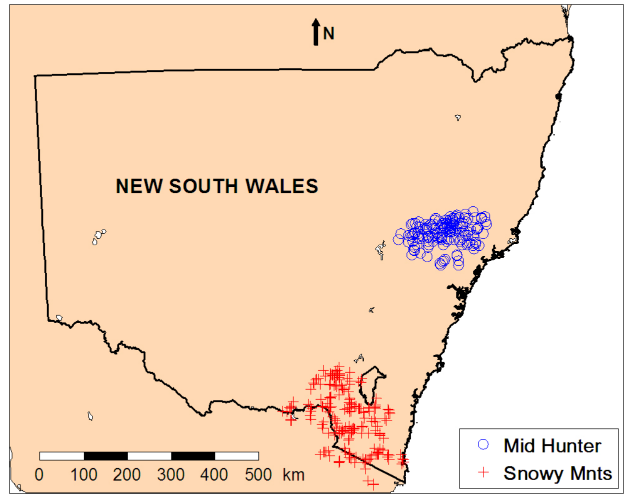

Figure 2: Location of the two study areas in NSW.

5 Study areas and datasets

The relative performance of AUSGeoid09 in mountainous regions was evaluated in two study areas located in NSW (Fig. 2). Both study areas represent typical mountainous terrain conditions encountered in Australia and exhibit large differences in elevation. NSW Land and Property Information (LPI) provided two GNSS network datasets collected over many years, together encompassing 186 survey marks with known AHD heights of sufficient quality (Class C Order 3 or better) on public record in the Survey Control Information Management System (SCIMS). SCIMS is the state's database containing about 250,000 survey marks across NSW, including coordinates, heights and metadata [21]. For a discussion of the terms class and order, the reader is referred to [20] and [22]. While it is acknowledged that [20] has recently been superseded by [23], this update does not affect the outcome of the analysis presented in this paper.

The Mid Hunter GNSS network adjustment covers an area of approximately 13,000 km^2, stretching from about 115 km south of the Mount Royal National Park to 170 km east of Mudgee. The terrain is mainly composed of valleys and mountains with elevations ranging between 20 m and 1,400 m. This dataset consists of 327 independent GNSS baselines observed between 147 marks. Of these, 82 SCIMS marks have known AHD heights (C3 or better), including 40 spirit-levelled marks of classification LCL3 or better.

The Snowy Mountains GNSS network adjustment covers an area of about 35,000 km^2, approximately bounded in the north by Tumut and the ACT border to Cooma, and in the south by Albury and the Victorian border towards the coast. The terrain exhibits an undulated topography composed of mountains reaching a peak of 2,228 m and low valleys with elevations of about 200 m. The GNSS dataset consists of 629 independent baselines observed between 263 marks. Of these, 104 SCIMS marks have known AHD heights (C3 or better), including 94 spirit-levelled marks of classification LCL3 or better.

In total, across both study areas, this provided 186 checkpoints with known AHD heights of sufficient quality for a practical AUSGeoid09 performance verification in relative terms and a comparison to AUSGeoid98. It should be noted that while some of the GNSS data used in this study contributed to the generation of AUSGeoid09, the datasets are considered sufficiently independent for the purpose of this study.

6 Data processing and analysis

The two GNSS networks used in this study were subject to several adjustments performed using the GeoLab least squares adjustment software. These adjustments were constrained to the national datum (GDA94) by holding several AUSPOS solutions [24] fixed, i.e. 7 and 27 marks in the Mid Hunter and Snowy Mountains networks respectively. N values were computed based on bi-cubic interpolation, using both AUSGeoid09 and AUSGeoid98 to enable comparison between the two models. The resulting GNSS-derived AHD heights are therefore independent of the official AHD heights on public record.

Both the Mid Hunter and the Snowy Mountains networks generated Class A surveys as per [20]. The ellipsoidal heights for the 104 checkpoints in the Snowy Mountains network displayed an average uncertainty of

±0.016 m (1 sigma), or ±0.031 m at the 95% confidence interval (CI). The Mid Hunter network performed slightly better, resulting in an average uncertainty of ±0.012 m (1 sigma), or ±0.024 m (95% CI), for the 82 checkpoints.

6.1 Comparison over all observed baselines

In the Mid Hunter network, analysis of the 104 observed baselines shows that AUSGeoid09 and its predecessor AUSGeoid98 perform at the same level as far as recovering AHD heights from GNSS is concerned, with an RMS of about 0.046 m. AUSGeoid09-derived results included no outliers and one outlier was identified when using AUSGeoid98. On average, AUSGeoid98 actually performs slightly better when the average baseline length of 13.1 km is considered (3.7 ppm vs. 4.0 ppm). However, this 0.3 ppm difference is equivalent to less than 4 mm in height over the average baseline length, i.e. less than 10% of the allowable misclose. Both models perform similarly in meeting 3rd order differential levelling specifications, with about 65% of baselines within specifications. Interestingly, the highest AUSGeoid98 residuals occur at different elevation ranges without any particular pattern, while the highest AUSGeoid09 residuals occur for baselines with height differences between 400 m and 750 m. However, it should be noted that this finding is based on only 6 out of 104 observed baselines and not evident when all possible mark-to-mark combinations are investigated (cf. Table 1).

In the Snowy Mountains network, analysis of the 66 observed baselines demonstrates that AUSGeoid09 performs moderately better than its predecessor AUSGeoid98. AUSGeoid09-derived results showed no outliers, while one outlier was identified when using AUSGeoid98. The RMS dropped from 0.064 m to 0.051 m, resulting in an improvement factor of almost 1.3. AUSGeoid09 also performs slightly better when the average baseline length of 7.5 km is considered (7.9 ppm vs. 8.1 ppm) – this 0.2 ppm difference is equivalent to only 1.5 mm in height over the average baseline length (less than 5% of the allowable misclose). AUSGeoid09 produced better results in regards to meeting 3rd order differential levelling specifications, with about 61% vs. 56% of baselines within the maximum allowable misclose.

6.2 Comparison over all possible mark-to-mark combinations

In the Mid Hunter network, a total of 3,320 possible mark-to-mark vector combinations were analysed. Although AUSGeoid09-derived results identified 15 outliers (none were identified when AUSGeoid98 was applied), it is evident that AUSGeoid09 is far superior to its predecessor. The RMS dropped from 0.105 m to 0.056 m, i.e. an improvement factor of 1.9. Considering the average vector length of 74.9 km, using AUSGeoid09 improved performance by a factor of 1.7, from 1.5 ppm to 0.9 ppm. This 0.6 ppm difference translates into 45 mm in height over the average vector length, thus having a substantial effect (43% of the allowable misclose). Furthermore, 90% of the AUSGeoid09-derived residuals met 3rd order differential levelling specifications (Fig. 3a), while only 61% of AUSGeoid98-derived residuals achieved the same (Fig. 3b).

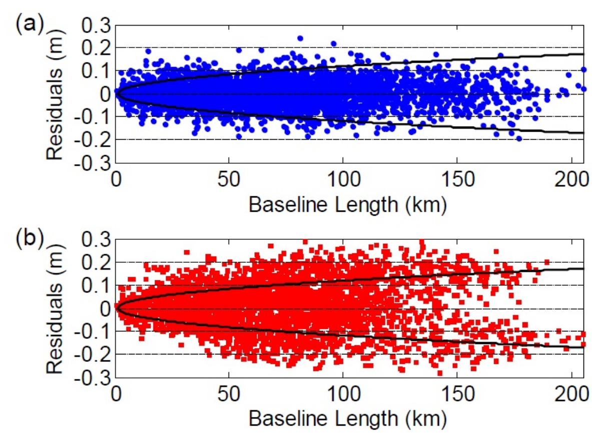

Figure 3: Mid Hunter residuals between (a) AUSGeoid09- and (b) AUSGeoid98-derived and official AHD height differences over 3,320 possible vectors. The unbroken line describes the allowable 3rd order differential levelling misclose.

In the Snowy Mountains network, a total of 5,356 possible mark-to-mark combinations were analysed. Again, although AUSGeoid09-derived results identified 15 outliers (3 were identified when using AUSGeoid98), it is evident that AUSGeoid09 performs much better than AUSGeoid98. The RMS dropped from 0.127 m to 0.099 m, translating into an improvement factor of 1.3. Considering the average vector length of 113.4 km, AUSGeoid09 improved performance from 1.4 ppm to 1.1 ppm. This 0.3 ppm difference is equivalent to 34 mm in height over the average vector length, i.e. 27% of the allowable misclose. Furthermore, 77% of the AUSGeoid09-derived residuals met 3rd order differential levelling specifications (Fig. 4a), while only 64% of AUSGeoid98-derived residuals fell within these bounds (Fig. 4b).

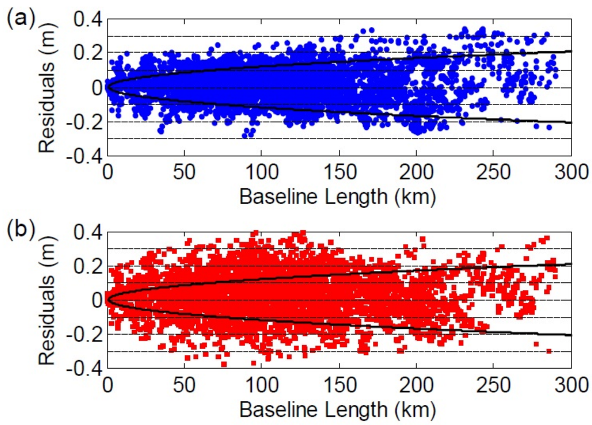

Figure 4: Snowy Mountains residuals between (a) AUSGeoid09- and (b) AUSGeoid98-derived and official AHD height differences over 5,356 possible vectors. The unbroken line describes the allowable 3rd order differential levelling misclose.

6.3 Comparison over all possible mark-to-mark combinations up to 100 km in length

The comparison over all possible mark-to-mark combinations (section 6.2) includes long vectors that are often well above 100 km in length. Generally speaking, it is unlikely that GNSS users perform network adjustments with baselines of this length, unless they contribute to state-wide or national control networks. Therefore, this third test was conducted to verify the performance of AUSGeoid09 based on the subset of all possible vectors up to 100 km in length, providing a more realistic GNSS network performance evaluation approach from a practical point of view.

In the Mid Hunter network, a total of 2,526 possible vectors with lengths up to 100 km were analysed. Although AUSGeoid09-derived results identified 12 outliers (none were identified when AUSGeoid98 was applied), AUSGeoid09 again shows superior performance to AUSGeoid98. The RMS dropped from 0.095 m to 0.055 m, i.e. an improvement factor of 1.7. Considering the average vector length of 57.4 km, using AUSGeoid09 improved performance from 1.6 ppm to 1.1 ppm (improvement factor of 1.5). This 0.5 ppm difference translates into more than 28 mm in height over the average vector length, thus having a substantial effect (31% of the allowable misclose). Moreover, 87% of the AUSGeoid09-derived residuals met 3rd order differential levelling specifications, while only 61% of AUSGeoid98-derived residuals achieved the same.

In the Snowy Mountains network, a total of 2,361 possible vectors up to 100 km length were analysed. For both models, a similar number of outliers was identified (7 vs. 6). Again, it is evident that AUSGeoid09 performs much better than AUSGeoid98, with the RMS dropping from 0.122 m to 0.082 m (improvement factor of 1.5). Considering the average vector length of 60.9 km, AUSGeoid09 improved performance from 2.3 ppm to 1.7 ppm. This 0.6 ppm difference is equivalent to more than 36 mm in height over the average vector length, again having a substantial effect (38% of the allowable misclose). In this case, 71% of the AUSGeoid09-derived residuals met 3rd order differential levelling specifications, compared to only 53% of AUSGeoid98-derived residuals falling within specifications.

6.4 Comparison over all possible mark-to-mark combinations as a function of AHD height difference

In mountainous regions, it is useful to investigate the performance of the two quasigeoid models as a function of AHD height, or height difference in relative terms. While it is recognised that the sample of checkpoints (and therefore mark-to-mark combinations) decreases considerably with increasing elevation, this will provide an indication of how well the two models fit AHD in undulating terrain. Following the approach taken by [14], Figure 5 illustrates the residuals for both quasigeoid models as a function of AHD height difference for the 3,320 vectors located in the Mid Hunter study area. Figure 6 shows the corresponding results for the 5,356 baselines in the Snowy Mountains study area.

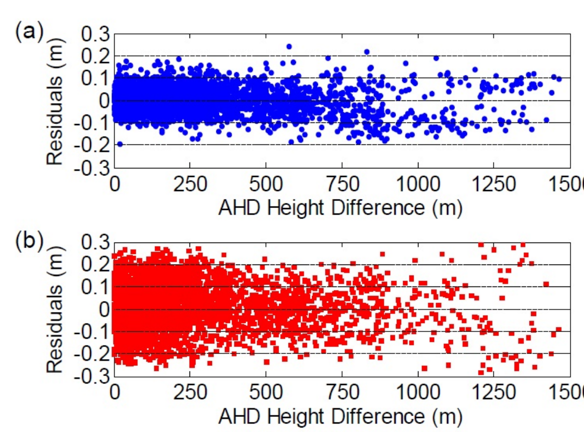

Figure 5: Mid Hunter residuals between (a) AUSGeoid09- and (b) AUSGeoid98-derived and official AHD height differences over 3,320 possible vectors as a function of AHD height difference.

Table 1: Mid Hunter verification: Descriptive statistics of the residuals between AUSGeoid09-derived and official AHD height differences as a function of AHD height difference.

AHD Height Diff (m)	No. of Baselines	Min (m)	Max (m)	Mean (m)	Range (m)	STD (m)	RMS (m)
all data	3,320	−0.194	0.243	−0.001	0.437	0.056	0.056
0 – 200	1,764	−0.194	0.175	0.001	0.369	0.047	0.047
200 – 400	770	−0.170	0.185	0.005	0.356	0.055	0.056
400 – 600	358	−0.185	0.243	−0.003	0.428	0.060	0.060
600 – 800	204	−0.175	0.189	−0.016	0.363	0.067	0.069
800 – 1,000	118	−0.185	0.219	−0.034	0.404	0.091	0.097
1,000 – 1,200	54	−0.161	0.190	−0.047	0.351	0.094	0.105
1,200 – 1,400	47	−0.133	0.118	0.015	0.251	0.079	0.079
> 1,400	5	−0.086	0.104	0.045	0.190	0.078	0.083

Table 2: Mid Hunter verification: Descriptive statistics of the residuals between AUSGeoid98-derived and official AHD height differences as a function of AHD height difference.

AHD Height Diff (m)	No. of Baselines	Min (m)	Max (m)	Mean (m)	Range (m)	STD (m)	RMS (m)
all data	3,320	−0.284	0.287	0.009	0.570	0.105	0.105
0 – 200	1,764	−0.266	0.269	0.007	0.535	0.107	0.107
200 – 400	770	−0.250	0.269	0.023	0.518	0.102	0.104
400 – 600	358	−0.239	0.225	0.004	0.464	0.087	0.087
600 – 800	204	−0.223	0.206	−0.005	0.430	0.094	0.094
800 – 1,000	118	−0.217	0.247	0.012	0.464	0.102	0.102
1,000 – 1,200	54	−0.264	0.244	−0.005	0.508	0.110	0.109
1,200 – 1,400	47	−0.284	0.287	−0.055	0.570	0.187	0.193
> 1,400	5	−0.208	0.109	−0.100	0.317	0.128	0.152

Table 3: Snowy Mountains verification: Descriptive statistics of the residuals between AUSGeoid09-derived and official AHD height differences as a function of AHD height difference.

AHD Height Diff (m)	No. of Baselines	Min (m)	Max (m)	Mean (m)	Range (m)	STD (m)	RMS (m)
all data	5,356	−0.282	0.343	0.012	0.625	0.099	0.099
0 – 200	1,500	−0.278	0.343	0.012	0.621	0.093	0.094
200 – 400	1,087	−0.282	0.284	0.010	0.567	0.084	0.084
400 – 600	943	−0.260	0.293	0.006	0.553	0.110	0.110
600 – 800	901	−0.268	0.276	0.007	0.544	0.112	0.113
800 – 1,000	471	−0.209	0.300	0.017	0.509	0.104	0.106
1,000 – 1,200	237	−0.181	0.335	0.028	0.516	0.087	0.091
1,200 – 1,400	104	−0.169	0.331	0.021	0.500	0.091	0.093
1,400 – 1,600	66	−0.117	0.284	0.048	0.401	0.095	0.106
1,600 – 1,800	14	−0.110	0.151	−0.017	0.261	0.081	0.080
1,800 – 2,000	24	−0.090	0.243	0.041	0.333	0.082	0.091
> 2,000	9	−0.039	0.110	0.024	0.149	0.045	0.049

It is confirmed that AUSGeoid09 produces a smaller scatter or variation in the residuals and generally provides a better fit (i.e. residuals closer to zero). For higher elevation changes, this is particularly evident in the Snowy Mountains study area, clearly showing the improvement obtained when using AUSGeoid09.

These findings are supported by investigating descriptive statistics of the residuals between GNSS-derived and official AHD height differences, calculated for all possible

Table 4: Snowy Mountains verification: Descriptive statistics of the residuals between AUSGeoid98-derived and official AHD height differences as a function of AHD height difference.

AHD Height Diff (m)	No. of Baselines	Min (m)	Max (m)	Mean (m)	Range (m)	STD (m)	RMS (m)
all data	5,356	−0.380	0.389	0.024	0.769	0.124	0.127
0 – 200	1,500	−0.380	0.360	0.020	0.740	0.122	0.123
200 – 400	1,087	−0.336	0.351	0.020	0.687	0.124	0.126
400 – 600	943	−0.371	0.389	0.021	0.761	0.127	0.129
600 – 800	901	−0.326	0.353	0.017	0.679	0.123	0.124
800 – 1,000	471	−0.280	0.345	0.028	0.624	0.116	0.119
1,000 – 1,200	237	−0.286	0.346	0.058	0.633	0.104	0.119
1,200 – 1,400	104	−0.311	0.281	0.045	0.592	0.119	0.127
1,400 – 1,600	66	−0.297	0.378	0.123	0.675	0.147	0.191
1,600 – 1,800	14	−0.266	0.199	−0.019	0.465	0.151	0.147
1,800 – 2,000	24	−0.137	0.388	0.168	0.525	0.142	0.218
> 2,000	9	−0.329	0.376	0.013	0.705	0.283	0.267

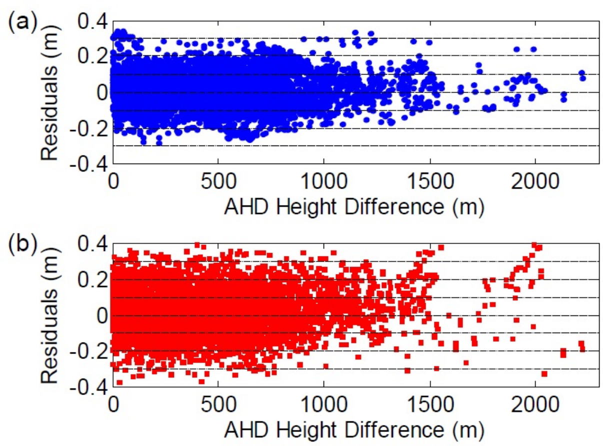

Figure 6: Snowy Mountains residuals between (a) AUSGeoid09- and (b) AUSGeoid98-derived and official AHD height differences over 5,356 possible vectors as a function of AHD height difference.

vectors in increments of 200 m in AHD height difference. Tables 1 and 2 show the statistics of the Mid Hunter network, while Tables 3 and 4 present the statistics of the Snowy Mountains dataset for the AUSGeoid09 and AUSGeoid98 models, respectively.

In the Mid Hunter network, both quasigeoid models demonstrate relatively stable and consistent sets of statistics with increasing height differences. RMS values for AUSGeoid09 slowly increase with increasing height difference, and the largest values occur for baselines with height differences of 800 m to 1,200 m. RMS values for AUSGeoid98 are larger but remain consistent until increasing considerably for elevation changes above 1,200 m. AUSGeoid09 shows substantial improvements in the standard deviation, RMS and the range of residuals across all height increments when compared to AUSGeoid98.

In the Snowy Mountains network, both quasigeoid models also present relatively stable and consistent statistics. RMS values for AUSGeoid09 show no evidence of deterioration with increasing elevation change. However, AUSGeoid98-derived results noticeably deteriorate for elevation changes above 1,400 m, indicating a much improved fit to AHD when using AUSGeoid09 for GNSS-based height transfer in heavily undulating terrain. Across all height increments, AUSGeoid09 again shows substantial improvements over AUSGeoid98 in the standard deviation, RMS and the range of residuals.

7 Concluding remarks

This paper has investigated the relative performance of the AUSGeoid09 quasigeoid model in mountainous terrain from a user's perspective and compared it to its predecessor AUSGeoid98. Two extensive datasets located in New South Wales were examined and analysed in a relative sense, i.e. using height differences based on GNSS-derived AHD heights and official AHD heights on public record.

As expected, AUSGeoid09 has demonstrated increased consistency and accuracy for GNSS-based height transfer compared to its predecessor, owing to the inclusion of a geometric component, a larger amount of input data and its higher density. While AUSGeoid09-based processing identified more outliers, this can be explained by standard deviations generally being substantially smaller than those for the AUSGeoid98-derived results. The graphical representation showed that AUSGeoid09 residuals are better distributed and consistently smaller than AUSGeoid98 residuals, which is particularly evident for large height differences.

It was found that AUSGeoid09 generally provides AHD height differences at the ±0.05 m to ±0.09 m level (1 sigma) in the two study areas. Importantly, for practicing surveyors, the use of AUSGeoid09 has substantially increased the percentage of GNSS-derived height differences meeting 3rd order differential levelling specifications, with up to 90% of AUSGeoid09-derived height difference residuals falling within the maximum allowable misclose. This is a very encouraging result, considering the well-known difficulties of spirit levelling in mountainous terrain and the increasing popularity of GNSS-based height transfer in practice.

It can be expected that future AUSGeoid models will provide further improvements for GNSS users, owing to improved modelling (including terrain modelling) and larger input datasets. To this end, for example, LPI is currently collecting extended GNSS datasets on levelled marks across NSW.

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