MANUAL
of the
NEW SOUTH WALES INTEGRATED SURVEY GRID

NEW SOUTH WALES
DEPARTMENT OF LANDS,
SYDNEY January, 1976

ERRATA

As at 1st May 1993, the following errors have been identified in the *'Manual of the NEW SOUTH WALES INTEGRATED SURVEY GRID' (publ. 1976)*.

1. Page 15, last line of text should read :
'are given in **Table I below and Annexure K - Zone Diagram.**'

2. Page 24, part 5.8
In formula (5.8), '0.6' should be '**0.06**' and add '}' at the end.
Formula (5.8) should read:

$$K = 0.99994 \left\{ 1 + 0.3081 \left[(E_1 . 10^{-7}) + (E_2 . 10^{-7}) - \mathbf{0.06} \right]^2 \right\}$$

3. Page 32, part 7.5
1/sin 1" is missing from the formula for E_5.
The second line of the formula should read:

$$+ \ 72 \frac{\nu'}{\rho'} t'^2 \ + \ 24 t'^4 \ \Big] \ . \ \frac{1}{\sin 1''}$$

4. Page 35, **lines 11 to 17** should be replaced by the following lines :

Assuming that

$$E - E_0 = \Delta E \quad \text{and} \quad \Delta E . 10^{-5} = (y)$$
$$N - N_0 = \Delta N \quad \text{and} \quad \Delta N . 10^{-5} = (x)$$

where E_0 and N_0 are I.S.G. co-ordinates for the adopted point on the zone boundary with latitude ϕ_0. New co-efficients K_3 to K_6 are calculated as follows:

$$k_1(y) - k_2(x) + k_3 = K_3 \qquad k_1(x) + k_2(y) + k_4 = K_4$$
$$K_3(y) - K_4(x) + k_5 = K_5 \qquad K_3(x) + K_4(y) + k_6 = K_6$$

5. Page 50, part 10.7
The last three lines should read :

$$2A = 2\ 624\ 375.50 = 2\ 624\ 375.5$$
$$A = 1\ 312\ 187.75 m^2$$
$$= \underline{131.2188 \text{ Ha}}$$

6. Page 53, in the table beneath Figure 5
The value in line *B* column *x* should be +70.6. The line should read :

B	230 15 15.1	155 04	+.42156	15 110	+26'.0	+70'.6	230 16 25.7

7. Page 62, Miscellaneous calculation (3)
The last line of the Scale Factor calculation should read :

$$= (1 - 0.000\,060\,000)(1 + 0.000\,187\,426) = 1 + .000\,127\,415$$

8. Page 91, sixth line from bottom of page
Replace the word '**taping**' with '**distance measurement**'.

9. Page 92, fifth line from bottom of page should read :
'... much the same **way** as he ...'

10. Page 93, figure 12
At the top of the diagram, after 'Number of errors' on the ordinate, delete '- v'.
At the bottom of the diagram, change 'Size of errors -v' on the abscissa to read 'Size of errors (v)'.

11. Page 122, fourth line from bottom of page should read :
'feature **of most** small calculators will automatically take this into account.'

12. Page 145 & 146 - In item '**Transverse Mercator Projection (TM)**'
On page 145, in line 2 of this definition delete '**(See Fig 20A)**'.
On page 146, in line 1, insert '**(See Fig 20A)**' after '*central meridian*'.

13. Page 154, Annexure A
In the last column (Metres), the third last figure, **1943** should be **1948**.

MANUAL

of the

NEW SOUTH WALES INTEGRATED SURVEY GRID

ERRATA

Two errors have been noted in this manual.

The first is contained on page 24, part (5.8) where the last figure in the formula K is shown as 0.6 when it should be 0.06.

The correct formula should read:

$$K=0{\cdot}99994\{1+0{\cdot}3081[(E_1.10^{-7})+(E_2.10^{-7})-0{\cdot}06]^2$$

The second error is on page 50 article 10.7 and occurs in the last three lines which should read:

$$2A = 2\,624\,375.50 = 2\,624\,375.5$$
$$A = 1\,312\,187.75 \text{ m}^2$$
$$= 131.2188 \text{ Ha}$$

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PREFACE

The introduction of the Integrated Survey Grid (I.S.G.), as the first comprehensive system of survey in this State, has not been without difficulty. The initial representations for the introduction of a co-ordinated statewide system commenced in the last century. Spasmodic attempts have been made throughout the years to introduce a proper system of survey. The Survey Co-ordination Act, 1949 was the first positive step towards the establishment of control surveys and connection to the Trigonometrical Survey. It is unfortunate that so little progress was made in implementing the provisions of this Act.

The first positive move towards the establishment of a modern and comprehensive system followed a request by the Institution of Surveyors, Australia, New South Wales Division at the instigation of the President, Mr I. C. Booth. With ministerial approval, a committee of investigation was established in 1968 to investigate and report on the introduction of an Integrated Survey System. The committee comprised the Registrar General and representatives of his Department, the chief or principal surveyors of twelve government departments and instrumentalities involved in surveying and representatives of the N.S.W. Division of the Institution and Universities of Sydney and New South Wales under the chairmanship of the Surveyor General. The committee was also assisted by numerous technical experts.

The committee undertook a detailed study, including investigation of overseas practices, particularly where similar changes had been introduced in recent years. Quite a number of the committee members had the benefit of experience in various survey systems in other countries. The committee reported to the Minister for Lands in July, 1969, and, in January, 1970, the Government approved the introduction of an Integrated Survey System and authorized the Surveyor General to commence the marking and control surveys and prepare draft legislation.

In November, 1971, the profession undertook the "Sydney Pilot Survey" in which a survey to establish some sixty-six control points, located in the commercial heart of Sydney, was undertaken by eighteen separate survey parties. This one-day exercise was designed to demonstrate the machinery and potential of integration. As the report indicated, it was successful in every way.

A recommendation for draft legislation was furnished to the Minister for Lands in April, 1972, and a copy of the draft provided to the Institution of Surveyors. Due to various representations, the Minister of Lands arranged for Sir John Overall to conduct a public inquiry to determine whether a more sophisticated system of survey was necessary and for report generally on the merits of the recommendations of the Investigation Committee.

Sir John Overall, in a report of August, 1974, recommended that the Integrated Survey System be introduced "forthwith" and that basic control be established in the urban and developing areas of the State in the first 5 years and in the other areas in the following 5 years.

A revised draft Bill has since been prepared on the lines recommended by Sir John Overall and it is hoped that it will be introduced in Parliament at an early date.

The Investigating Committee considered that the advantages to the State in establishing an Integrated System of survey can be summarized as:

(1) The establishment of one correlated system for all surveys.

(2) The system will provide a more positive basis for land titles and allow easier and less costly redefinition of these boundaries in the future.

(3) Abundant basic control for all mapping, surveys and compilation of cadastral maps and plans would be available.

CONTENTS

PART 1. DEFINITION AND APPLICATIONS OF THE INTEGRATED SURVEY GRID.

1. INTRODUCTION

1.1 *Integration*

The reasons for introducing an Integrated Survey system in New South Wales have been set out briefly in the preface. In establishing the system, one of the intentions has been to enable survey methods to be transposed into the new system with a minimum of change. There are three essential changes:

- surveys in Integrated Survey areas are based on control survey marks;
- projection corrections are applied to survey data to convert to projection data, before computation; and
- there is a new set of criteria for acceptable accuracy of survey data.

The corrections required are the *reduction to sea level*, the projection corrections: *scale correction*, the *arc-to-chord correction* and the *grid convergence*. In many surveys, perhaps the majority of cases, the corrections are negligibly small. Formulae for calculation of the corrections, with examples, are given in paragraphs 4.6 (sea level), 5.8 (scale correction), 5.5 (arc-to-chord) and 5.6 (grid convergence).

Most users of the Integrated Survey system will wish to have more detailed information on the system and its applications. This will enable them to make use of the positive advantages of working within a control survey system and applying co-ordinates in all computations. The *Manual* aims to fill the needs of these users. The requirement of surveys of high precision are specialized, and while the *Manual* provides full details on calculating the projection corrections to high precision, it does not deal with the methods of precise surveying.

Methods of calculation in the *Manual* are appropriate to programmable and non-programmable calculators of the pocket or desk-top types. Because the subject of electronic computing is specialized and subject to rapid changes, it is not covered in the *Manual*.

1.2 *Scope of the Manual*

The aims of this *Manual* are—

- to describe the co-ordinate system to be used for survey integration;
- to define standard symbols, terms and formulae for use in survey practice;
- to indicate methods of calculating projection corrections, which are convenient and appropriate for various orders of accuracy;
- to provide numerical examples for the standardisation and simplification of computations;
- to give examples of surveys based on co-ordinated survey control marks in an integrated system;
- to describe the accuracy standards introduced with survey integration and to indicate how they can be applied;
- to indicate methods for the determination of azimuth by astronomical methods;
- to describe the Australian Height Datum and indicate the reasons why it was chosen to supersede the State Standard Datum;
- to set out revised survey directions on determination of mean high water mark.

A section at the end of the *Manual* contains a glossary, the bibliography and annexures. The glossary is intended to provide simple descriptions of terms used in the *Manual* and in discussions on survey integration. These descriptions are not rigorous definitions.

Very few references are quoted in the *Manual*. However the bibliography provides a list of publications for further reading, including those referred to in the text. The bibliography is arranged according to the parts of the *Manual*. The annexures include tables and diagrams for easy determination of projection corrections, forms and tables for computation of coordinate transformations, and maps showing the levelling net and the integrated survey grid zones in New South Wales.

2. FIGURE OF THE EARTH AND GEODETIC DATUM

2.1 *Introduction*

The Australian National Spheroid (*ANS*) and *Australian Geodetic Datum* have previously been adopted for the computation of surveys in Australia. They are also adopted for the computation of Integrated Surveys in New South Wales.

2.2 *Australian Geodetic Datum.*

The *Australian Geodetic Datum* was defined by the 1966 Adjustment of first order triangulation of Australia, in which the fundamental station was the Johnstone Geodetic Origin.

In 1972, the first order network between Newcastle, Sydney, Wollongong and Canberra was extended towards the coast to include trigonometrical stations in the proclaimed Integration Survey areas. This network was readjusted retaining the 1966 values for the western peripheral stations. It is intended to retain the co-ordinates obtained in the 1966 adjustment and in the 1972 readjustment as noted above for the adjustment of second and lower order triangulation and traverse networks.

Any values obtained by future readjustment of first order triangulation and used in integrated surveys should be qualified by a statement of their source and used with the approval of the Surveyor-General only. No such statements are required for the 1966-1972 values.

2.3 *The Australian National Spheroid.*

The defining parameters of the Australian National Spheroid are:

Major semi-axis, $a = 6\,378\,160$ metres

Flattening, $f = 1/298.25$

Derived functions are:

Flattening, f $= 0.00335\,28918\,69$

Minor semi-axis, $b = a(1 - f)$ $= 6\,356\,774.719$ metres

$e^2 = 2f - f^2 = (a^2 - b^2)/a^2$ $= 0.00669\,45418\,55$

$e'^2 = e^2 + e^4/(1 - e^2) = (a^2 - b^2)/b^2$ $= 0.00673\,96607\,96$

$c = a/(1 - e^2)^{\frac{1}{2}}$ $= 6\,399\,617.255$ metres.

For the computation of radii of curvature in latitude ϕ, where ρ is the radius of curvature in the meridian and ν is the radius of curvature in the prime vertical:

$$V^2 = 1 + e'^2 \cos^2 \phi = \nu/\rho$$

$$\rho = c/V^3;\ \nu = c/V;\ R = (\rho\nu)^{\frac{1}{2}} = c/V^2$$

The following values are useful:

sin 1″ = 0.00000 48481 36811 1
π = 3.14159 26536
1 radian = 57.29577 9513 degrees
= 3 437.74677 08 minutes
= 206 264.80625 seconds

Conversion factors:

1 yard = 0.9144 metres exactly
1 foot = 0.3048 metres exactly

2.4 *Integrated Survey Grid (I.S.G.)*

A Transverse Mercator Projection is used as the basis for the computation of co-ordinates on the Integrated Survey Grid.

Surveys are to be connected to the stations of the state survey control system. The Transverse Mercator co-ordinates of these stations are calculated from latitudes and longitudes on the Australian Geodetic Datum as defined in para 2.2.

The Integrated Survey Grid is defined as follows:

(1) The projection is the Transverse Mercator Projection.

(2) Zones are 2° wide with $\frac{1}{4}$° overlaps.

(3) The true origin of each zone is the intersection of the Central Meridian with the Equator.

(4) A central scale factor $k_0 = 0.99994$ is applied to co-ordinates on the projection.

(5) Easting, E is defined by adding 300 000 metres to the value of y measured from the central meridian.

(6) Northing, N is defined by adding 5 000 000 metres to the value of x measured from the equator. All values of x south of the equator are negative.

(7) The zones are numbered in relation to the 6° zones of the Australian Map Grid (A.M.G.). Each 6° zone is subdivided into three 2° sections, each of which is covered by one I.S.G. zone. The I.S.G. zone identification consists of two parts, the first part is the corresponding A.M.G. zone number, and the second part, separated by a slash, indicates the number of the subdivision, from 1 to 3, increasing eastwards. For example, the eastern sector of A.M.G. zone 55 (144°-150° E.) is covered by I.S.G. zone 55/3 which extends from 148°-150° E. and has central meridian 149° E. Details of I.S.G. zones are given in table I see annexure K.

TABLE I

Zones of the Integrated Survey Grid, New South Wales

I.S.G. Zone	Extent (excl. overlaps)	Central Meridian
54/2	140-142° E.	141° E.
54/3	142-144	143
55/1	144-146	145
55/2	146-148	147
55/3	148-150	149
56/1	150-152	151
56/2	152-154	153

(8) Co-ordinates are normally quoted in metres.

(9) Notation of Co-ordinates. In quoting or recording I.S.G. co-ordinates, the Easting is always placed before the Northing.

The following procedure is recommended for the listing of co-ordinates. At the head of any listing of co-ordinates the figures common to all co-ordinates are noted as constants. These constants, one for the Eastings and one for the Northings, are subtracted from all co-ordinates in the co-ordinate list and in the computations, in order to avoid carrying redundant digits.

A vertical dotted line or a gap may be placed between the third and fourth figures from the decimal point to assist in the alignment of figures, as indicated in the example below. The decimal points are preprinted on forms designed for the listing of co-ordinates.

	E metres	N
Constant	200 000	1 100 000
A5	101 155.16	115.15
B27	125 311.00	13 351.85
B39	100 101.11	2 295.69
C11	98 115.35	20 015.70

Truncation of co-ordinates. The number of decimal places quoted should be sufficient to allow calculation of bearings and distances from co-ordinates to an accuracy consistent with the requirements of the survey or the regulations.

It is the practice of the Department of Lands to record to 0.001 metres, co-ordinates for control points which have been adjusted by the method of least squares.

3. SYMBOLS

ϕ = Geodetic latitude, negative south of the equator.

ϕ_1, ϕ_2 = Latitude at points 1 and 2 respectively.

$\phi_m = (\phi_1 + \phi_2)/2$

$\Delta\phi = \phi_2 - \phi_1$

$\Delta\phi''$ = $\Delta\phi$ expressed in seconds of arc.

λ = Geodetic longitude measured from Greenwich, positive eastwards.

$\Delta\lambda = \lambda_2 - \lambda_1$

λ_0 = Geodetic longitude of a central meridian.

ω = Geodetic longitude measured from a central meridian, positive eastwards; $\omega = \lambda - \lambda_0$.

y = Co-ordinate perpendicular to the central meridian measured from the central meridian, positive eastwards.

x = Co-ordinate parallel to the central meridian measured from the equator, negative southwards.

$E = y + 300\,000$ metres = Easting.

$N = x + 5\,000\,000$ metres = Northing.

ρ, ν = Radii of curvature of the spheroid in the meridian and prime vertical respectively.

α = Azimuth, clockwise through 360° from true north.

β = Grid bearing, clockwise through 360° from grid north.
θ = Plane bearing, clockwise through 360° from grid north.
γ = Grid convergence, positive at points east of the central meridian and negative at points west of the central meridian.
δ = Arc-to-chord correction, with sign defined by the equations: $\theta = \beta + \delta = \alpha + \gamma + \delta$.
$\Delta\alpha$ = Meridian convergence.
s = Spheroidal distance.
S = Grid distance = plane distance.
d = Ground distance.
m = Meridian distance, true distance from the equator, negative southwards.
a, b = Major and minor semi-axes of the spheroid.
e^2 = $(a^2 - b^2)/a^2$ = (eccentricity)^2.
e'^2 = $(a^2 - b^2)/b^2$ = (second eccentricity)^2.
k_0 = Central scale factor = 0.99994 for I.S.G.
k = Point scale factor.
K = Line scale factor.
t = $\tan\phi$.
ϕ' = Foot point latitude. The latitude for which $m = x/k_0$
t', ρ', ν' are functions of the latitude ϕ'.
R^2 = $\rho\nu$.
r^2 = $R^2 k_0^2 = \rho\nu k_0^2$.

Note that y, x, E, N, S, r, k and K include the central scale factor, k_0; whereas d, s, ρ, ν, R and m are true distances, which must be specifically multiplied by k_0 when necessary.

4. SURVEY DATA ON SPHEROID.

Most of the quantities defined below are illustrated in figure 2.

4.1 *Azimuth*

Azimuth α, is a horizontal angle reckoned clockwise from the north direction of the spheroidal meridian at a point, from 0° to 360°.

It is observed astronomically (Astronomical Azimuth α_A) or computed from co-ordinates on the projection or spheroid (Geodetic Azimuth α_G or α).

The Laplace correction to an astronomically observed Azimuth is not normally applied except in first order surveys. In New South Wales the correction will seldom exceed 5 arc seconds.

4.2 *Geographic co-ordinates*

Geographic (geodetic) co-ordinates, latitude, ϕ, and lontidue, λ, are not directly used in Integrated Surveys and are required only for special computations.

Latitude is negative south of the equator and longitude is measured positive eastwards from Greenwich.

4.3 *Meridian convergence*

Meridian convergence, $\Delta\alpha$, is the change in azimuth of a line between two points on the spheroid.

Reverse azimuth = Forward azimuth + meridian convergence ± 180°.

4.4 *Mean radius of the earth*

For surveys in the integrated system it is sufficient to adopt $R_m = 6\ 370\ 100$ metres, which corresponds to a latitude of 34° and 45° azimuth. Changes in latitude and azimuth have negligible effect on the computation of projection corrections within the State of New South Wales.

Curvature functions in metres

The following values of curvature functions are useful in computations:

$r_m = 0.99994\ R_m = 6\ 369\ 700$
$10^{14}/2r_m^2 = 1.2323$
$10^{10}/2r_m^2 \sin 1'' = 25.419$
$10^{14}/6r_m^2 = 0.41078$
$10^{10}/6r_m^2 \sin 1'' = 8.4729$

4.5 *Ground distance*

Ground distance, d, is a horizontal measurement, at the mean elevation of the line, to which the mean sea level and projection corrections have *not* been applied.

4.6 *Spheroidal distance*

Spheroidal distance, s, is a sea-level distance on the spheroid.

If d is a measured distance at an average elevation of h (*see* para. 4.5 and figure 1) then the required mean sea-level correction is

$$s - d = -d.\frac{h}{R_m} \qquad \cdot\cdot(4.1)$$

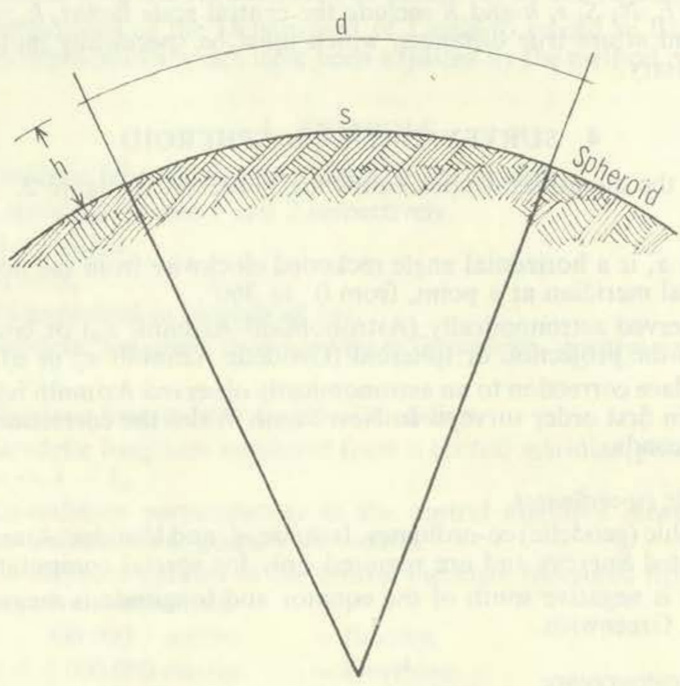


FIGURE 1—SEA-LEVEL CORRECTION
Distance d measured at elevation h above the spheroid, is longer than the spheroidal distance s

If an accuracy of 1 : 100 000 is required for the reduced length, the mean height should be correct to 60 metres (200 ft). The propagation of the error is linear, for example a change in height of 60 metres would cause a change of $d/100\,000$ in the reduced distance, s.

The quantity $10^6/R_m = 0.15698 = 0.157$ with an accuracy of better than 1 part per million (p.p.m.) in New South Wales latitudes, so that, for practical computation: $s - d = -0.157\,(d.10^{-3})\,(h.10^{-3})$.

The evaluation of this formula is simpler than it appears at first sight, the quantities in brackets being obtained by shifting the decimal point three places to the left in each case.

Example. Calculation of spheroidal distance s.

$d = 4\,000$ m; $h = 500$ m.

$s - d = 0.157(-4.0)(0.5) = -0.314$ m.

$s = 4\,000 - 0.314 = 3\,999.686$ (spheroidal distance).

The Annexures provide the corrections in convenient form:

The *factor s/d* is tabulated in annexure A.

Corrections may be obtained from the table in annexure B.

Combined corrections for sea level and scale may be obtained from the graph in annexure D.

5. SURVEY DATA ON INTEGRATED SURVEY GRID (I.S.G.)

5.1 *Introduction*

The quantities defined below are illustrated in figure 2.

The shortest line joining two points on the spheroid (the geodesic) projects on the Integrated Survey Grid as an arc and the projected grid length, S, is equal to the spheroidal length, s, approximately 70 km E or W from the central meridian. The grid length is shorter than the spheroidal length for lines closer to the central meridian, and longer than the spheroidal length for lines further from the central meridian.

Computations on the projection may be carried out using spheroidal bearings and distances. However, survey data on the spheroid may be reduced to the plane of the Integrated Survey Grid by the application of projection corrections, so that all computations may be effected by the formulae of plane trigonometry. This method has been adopted throughout the manual. This is far simpler and more convenient and is the method normally adopted.

The practical application of projection corrections is discussed in section 6.

5.2 *Grid North*

Grid North at any point on the projected geodesic arc is represented by a straight line parallel to the central meridian.

5.3 *Grid bearing*

Grid bearing, β, at a point on the projected geodesic arc is the angle measured clockwise from Grid North to the tangent to the arc at that point, from 0° to 360°. Grid bearing may be obtained by subtracting the arc-to-chord correction (*see* para. 5.5) from a plane bearing, or by adding grid convergence (*see* para. 5.6) to an azimuth.

$$\beta = \theta - \delta$$
$$\beta = \alpha + \gamma$$

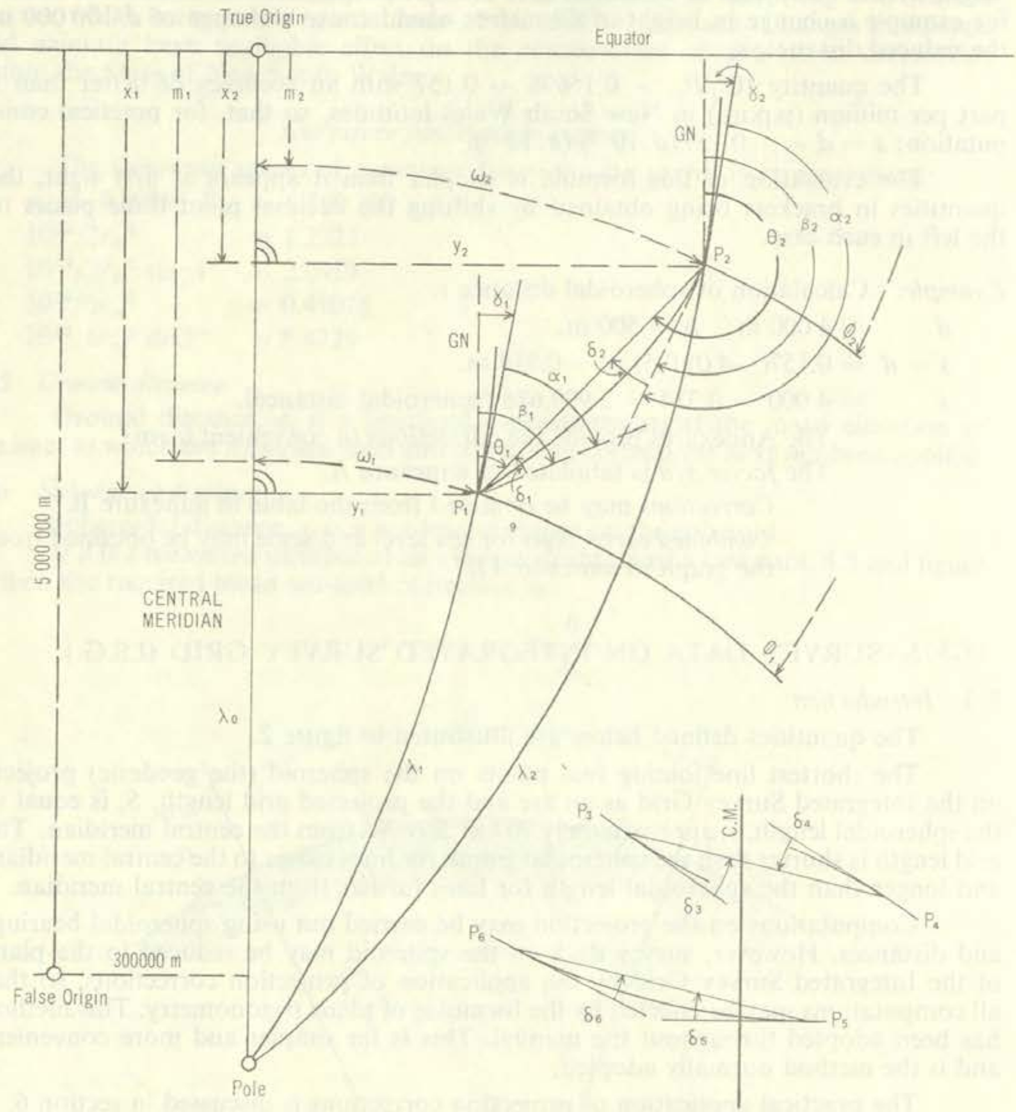


FIGURE 2—INTEGRATED SURVEY GRID

$P_1\,P_2$ = Line east of central meridian

$P_3\,P_4$, $P_5\,P_6$ = Lines crossing central meridian (*see* para 5.5)

GN — Grid North

The following quantities are negative in the figure

$\delta_1\ \delta_3\ \delta_4\ \delta_6\ \phi_1\ \phi_2\ x_1\ x_2\ m_1\ m_2$

5.4 *Plane bearing*

Plane bearing, θ, is the angle between Grid North and the straight line or chord joining the ends of the projected geodesic arc defined in para. 5.1.

A plane bearing may be obtained in one of the three following ways:

1. directly from grid co-ordinates by formulae

$$\tan\theta = \frac{E_2 - E_1}{N_2 - N_1} = \frac{\Delta E}{\Delta N}$$

or

$$\cot\theta = \frac{N_2 - N_1}{E_2 - E_1} = \frac{\Delta N}{\Delta E}$$

(*See* para. 10.1 for details of computation.)

2. by adding the arc-to-chord correction to a grid bearing.

$$\theta = \beta + \delta$$

3. by adding to a plane bearing an angle which has been reduced to the plane.

5.5 *Arc-to-chord correction*, δ

The arc-to-chord or $(t - T)$ correction is the angular quantity to be added algebraically to a grid bearing to obtain a plane bearing. It is the angle between the geodesic which projects as a curve, and the chord or straight line, between two points on the projection.

Thus any observed spheroidal direction or bearing is reduced to the plane of the projection by applying the angle δ.

The sign of the correction may be determined either from the formulae given below, or graphically by noting that the projected geodesic arc is always bowed away from the central meridian, (lines P_1P_2, P_3P_4 in figure 2).

If the line crosses a central meridian within one third of its length from one end, the bow is determined by the longer part (line P_5P_6). In this case δ is extremely small.

The arc-to-chord correction at the station occupied is given by:

$$\delta_{occ-obs} = \frac{(N_{occ} - N_{obs})(2y_{occ} + y_{obs})}{6r_m{}^2 \sin 1''}$$

where the subscript *occ* refers to the occupied station and the subscript *obs* refers to the observed station.

$$y = E - 300\,000 \text{ m}$$

For practical computation, using false Easting E:

$$\delta_{occ-obs} = 8''.5\,[(2E_{occ} \times 10^{-5}) + (E_{obs} \times 10^{-5}) - 9.00] \times [N_{occ} - N_{obs}] \times 10^{-5} \qquad \ldots(5.1)$$

Move the decimal point of E and ΔN by five places and round off to two places.

Example. Calculation of arc-to-chord corrections.

Data:	E_1	422 145.515	N_1	1 817 938.975
	E_2	398 112.145	N_2	1 828 011.324

When Point 1 is occupied and Point 2 is observed:

From (5.1) $\delta = 8''.5\,(2 \times 4.22 + 3.98 - 9.00)\,(-0.10)$

$= 8''.5\,(-0.342) = \underline{-2''.9}$

When Point 2 is occupied and Point 1 observed:

$\delta = 8''.5\,(2 \times 3.98 + 4.22 - 9.00)\,(0.10)$

$= 8''.5\,(0.318) = \underline{+2''.7}$

Generally the magnitudes of arc-to-chord corrections at both ends of a line are approximately the same, the signs of the corrections being opposite. The difference in magnitudes is—

$$\Delta\delta = 8''.5\,(E_{occ} - E_{obs})\,(N_{occ} - N_{obs}) \times 10^{-10}$$

irrespective of the distance from the Central Meridian. If $\Delta N = \Delta E = 15$ km, the difference in magnitudes is 0.2 arc seconds.

If E_m is an approximate mean Easting for the line the absolute value of

$$\delta = 25''.4\,(E_m.10^{-5} - 3.00)\,\Delta N \qquad \ldots(5.2)$$

Example. Calculation of arc-to-chord correction—simplified method

Data: As in *Example*, para. 5.5

$E_m = 410\,000$ m; $N = 10\,100$ m and the absolute value δ is, from (5.2):

$\delta = 25''.4\,(4.10 - 3.00) \times 0.10$

$= \underline{2''.8}$

Alternatively the *correction* may conveniently be read off the graph in annexure *D*, to the nearest half second.

5.6 *Grid convergence, γ*

The grid convergence γ at a point is the angle between grid north and the tangent to the arc of the meridian at that point.

Grid bearing = azimuth + grid convergence

$\beta = \alpha + \gamma$

Thus any observed astronomical azimuth may be reduced to a grid bearing by the application of γ.

In the southern hemisphere grid convergence is positive for points east of the central meridian, and negative west.

The grid convergence is most easily determined from geographic co-ordinates. If the longitude of the central meridian is λ_0 and that of a point in latitude ϕ is λ then, to the nearest arc second

$$\gamma = -(\lambda - \lambda_0) \sin\phi, \qquad \ldots(5.3)$$

Note that in this formula, ϕ is negative.

Using I.S.G. tables:

$\gamma'' = b_1.p$

where $p = 0.0001\,\omega$, $\qquad \omega = \lambda - \lambda_0$, in seconds of arc.

Alternatively, if I.S.G. co-ordinates in metres are known, an accuracy to the nearest arc second may be obtained by using tables for the conversion of Grid to Geographic co-ordinates as follows:

Select the term a_0 closest to the given Northing, note the corresponding latitude to the nearest minute and look up the term d_1 for that latitude, then

$$\gamma'' = d_1\,(y.10^{-6}) \qquad \ldots(5.4)$$

The same accuracy may be obtained by using the table in annexure C.

For high precision surveys, γ may be obtained to an accuracy of 0″.01 by interpolating latitude to 0.1 second and using the formulae given in the I.S.G. tables.

Example. Calculation of grid convergence.

1. To nearest second.

Data: $E_1 = 422\,145.515$ $N_1 = 1\,817\,938.975$

Calculation: $y = E - 300\,000$

$y = 122\,145$

$\phi = -28°\,45'$ from tables: $d_1 = 17\,729$

$\gamma = 0.12214\,5 \times 17\,729 = 2\,165''.5 = \underline{36'\,06''}$

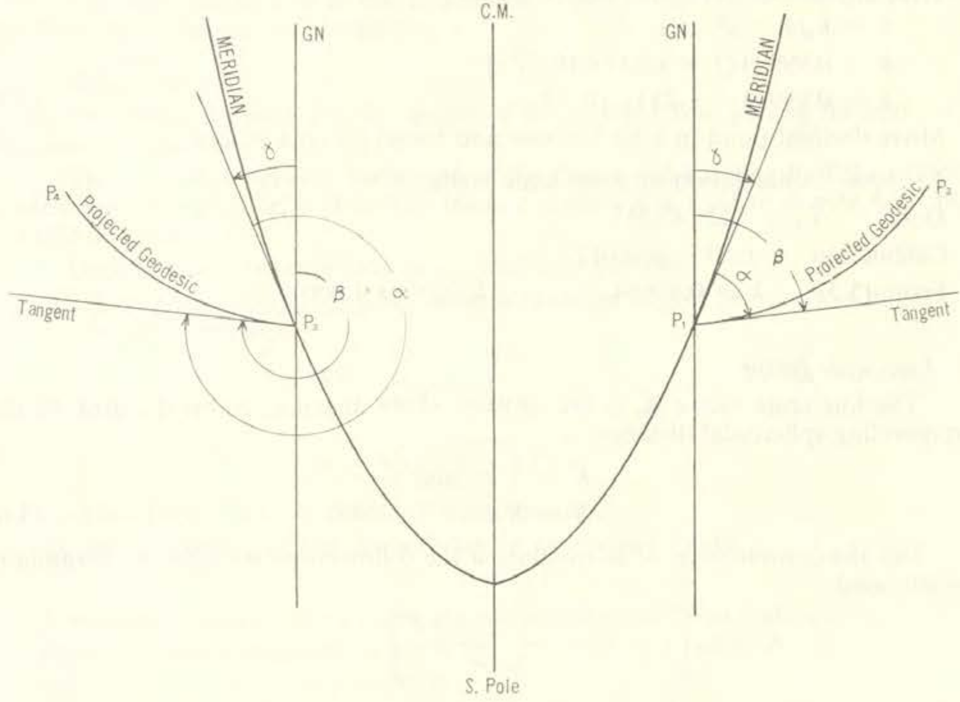


FIGURE 3—RELATIONSHIP BETWEEN AZIMUTH α, GRID CONVERGENCE γ AND PLANE BEARING β

2. To 0.01 seconds.

Interpolating in a_0: $(1\,818\,619 - 1\,817\,939)/30.785 = 22.1$

$\theta = -28°\,45'\,22''.1$

From tables $d_1 = 17\,729.23 + 22.10 \times 0.2036 = 17\,733.73$

$d_3 = 188.1$

$\gamma = d_1(y.10^{-6}) - d_3(y.10^{-6})^3$

$= 17\,733.73 \times 0.12214\,55 - 188.1\,(0.1221)^3$

$= 2\,166.09 - 0''.34 = 2\,165''.75 = \underline{36'\,05''.75}$

If the azimuth of the line from Point 1 to Point 2 in para. 5.5 is 292° 08′ 15″.6, then the grid bearing of the line is

$$\alpha + \gamma = \beta$$

$$292° \, 08' \, 15''.6 + 36' \, 05''.8 = 292° \, 44' \, 21''.4$$

and applying the arc-to-chord correction $\delta = -2.9$, the plane bearing θ is 292° 44′ 18″.5.

5.7 *Point scale factor*

The point scale factor k, at a point on the curve, is the ratio of an infinitesimal distance on the grid to the corresponding distance on the spheroid.

$k = dS/ds.$

With an accuracy of better than 1 p.p.m :

$k = k_0(1 + y^2/2r^2)$

$k = 0.99994 \, [1 + 1.23 \, (y.10^{-7})^2.]$

$k = 0.99994 + 1.23 \, (y.10^{-7})^2$ ...(5.5)

Move decimal point in y by 7 places and round off to 4 places.

Example. Calculation of point scale factor.

Data: $E_1 = 422\,145.515$

Calculation $y.10^{-7} = 0.0122$

From (5.5) $k = 0.99994 + 1.23 \, (0.0122)^2 = 1.00012\,3$

5.8 *Line scale factor*

The line scale factor K, is the ratio of plane distance, S, on the grid, to the corresponding spheroidal distance, s.

$$K = S/s \quad \text{and}$$

$$S = K.s \qquad \ldots(5.6)$$

For the computation of triangulation the following more accurate formula is generally used:

$$K = k_0 \left[1 + \frac{y_1^2 + y_1 \, y_2 + y_2^2}{6r_m^2} \right]$$

With an accuracy of 1 p.p.m. if $E = 31$ km, or 2 p.p.m. if $E = 44$ km, anywhere in the zone, the point scale factor for the midpoint of the line may be adopted as line scale factor:

$$K = k_0 \left[1 + \frac{y_m^2}{2r_m^2} \right] \text{ where } y_m = \frac{y_1 + y_2}{2} \qquad \ldots(5.7)$$

For practical computation, using false Eastings:

$$K = 0.99994 \{1 + 0.3081 \, [(E_1 . 10^{-7}) + (E_2 . 10^{-7}) - 0.6]^2 \qquad \ldots(5.8)$$

Move decimal point in E by 7 places and round off to 4 places.

Substitution $y_m = y$ in equation (5.5) may be used.

Example. Calculation of line scale factor.

Data: as in *Example*, para. 5.5

From (5.8) $K = 0.99994\,[1 + 0.3081\,(0.0422 + 0.0398 - 0.06)^2]$
$= 0.99994\,[1 + 0.3081(0.022)^2]$
$= 0.99994 \times 1.00014\,9$
$K = \underline{1.00008\,9}$

$y_m = (122\,145 + 98\,112)/2 = 110\,128;\ y_m . 10^{-7} = 0.011$
Using equation (5.5)
$K = 0.99994 + 1.23\,(0.011)^2 = \underline{1.00008\,9}$

Alternatively *scale factors* may be obtained from the table in annexure A and *scale corrections* from annexure B, to an accuracy of 1/100 000.

5.9 *Grid distance*

The grid distance S is the length measured along the arc of the projected geodesic whose spheroidal distance is s.

5.10 *Plane distance*

The plane distance S is the length of the straight line joining the ends of the projected geodesic arc.

The difference between the length of the chord and the length of the projected geodesic arc is negligible, and in this *Manual* symbol S is used to denote both plane and grid distances.

Grid or plane distance may be computed as follows.

(a) from grid co-ordinates after the computation of grid bearing θ as in para. 5.4 by the formulae:

$$S = \frac{\Delta E}{\sin\theta} = \frac{\Delta N}{\cos\theta}, \text{ or}$$

$$S = \sqrt{(\Delta E)^2 + (\Delta N)^2}$$

(*See* para. 10.1 for details of computation.)

(b) by application of line scale factor to spheroidal length

$$S = s . K \qquad \ldots(5.10)$$

Example. Calculation of plane distance from spheroidal distance

Data: as in *Example*, para. 5.5
$s = 26\,056.366$ m
$K = 1.00008\,9$ from *Example*, para. 5.8

From (5.10) $S = 26\,056.366 \times 1.00008\,9 = \underline{26\,058.685\text{ m}}$

6. APPLICATION OF PROJECTION CORRECTIONS

6.1 *Introduction*

This section gives details of the practical application of the corrections which must be applied to survey data before it can be used in calculations on the projection. Because the zone width of 2° was selected for the Integrated Survey Grid these corrections are always very small. The order of size is in fact smaller than the permissible accidental errors in normal property or engineering surveys. Paragraphs 6.2, 6.3 and 6.5 deal with the magnitude of each correction and indicate the most convenient method of determining and applying each, in the case of surveys of normal accuracy. Paragraph 6.6 deals with the case of surveys of higher precision.

Besides the application of projection corrections, distances are required to be reduced to sea level (equation (4.1)) and the example in para. 4.6 indicates how this correction is applied. Alternatively this correction can conveniently be combined with the projection scale correction using the graph of annexure D.

6.2 *Arc-to-chord correction*, δ

For normal property and engineering surveys the arc-to-chord correction is negligible. The following table with Eastings as argument gives the length of line in the north-south direction (ΔN) which requires a correction of one second.

TABLE II

North-South component of lines for arc-to-chord corrections of one second

Easting		ΔN for $\delta = \pm 1''$
West of C.M.	East of C.M.	metres
280 000 metres	320 000	20 000
260 000	340 000	10 000
240 000	360 000	6 500
220 000	380 000	5 000
200 000	400 000	4 000
180 000	420 000	3 000

For a line of any other bearing, δ is a function of the Easting of the point 1/3 of the distance along the line from the end at which the correction is required.

Determining the correction. The absolute value of δ may be estimated from the graph in annexure D and the correct sign either obtained from figure 2, or deduced from the information that the projected line of sight is always bowed away from the central meridian.

6.3 *Scale correction*

The maximum distortion of the spheroidal length is of the order of 1/16 670 within the zone and 1/8 000 on the edge of the $\frac{1}{4}°$ overlap. The correction is negligible for many surveys.

Determining the correction. Distances may be corrected by multiplying by the scale factor (k) determined from the critical table in annexure A, or by applying the correction calculated from the table in annexure B, which gives the correction per 1 000 units. Alternatively the combined scale and sea-level corrections on a length of 1 000 units or the factor can be determined by interpolation on the appropriate elevation graph of annexure D.

6.4 *Combined correction for scale, sea-level and temperature*

For measurements with a steel band or tape, the scale, sea-level and temperature corrections (SST corrections) can be combined. This is achieved by calculating a new "standard" temperature for the band, such that the temperature correction applied will automatically include the scale and sea-level corrections. This temperature is called the *SST reference temperature.* It will be different for various Eastings and elevations, but one value will be applicable over a considerable range. For example, if corrections are required to be accurate to 1/50 000, one value of the reference temperature covers an elevation range of 250 m (800 ft) and a range of Eastings of 16 km near the edge of the overlap or 70 km near the central meridian.

Example. Calculation of SST reference temperature for a steel band for combining sea-level and projection scale corrections with the temperature correction.

Data: Easting, 420 km. Elevation h, 470 m.
Steel band standard temp, 22° C.
Coefficient of expansion 0.000011/° C.

Calculation of corrections on a length of 100 m.

Scale factor, equation (5.7) $K = k_0 (1 + y^2/2r_m^2)$
$K = 1.000117$

Correction on 100 m	$= + 0.0117$
Sea-level, equation (4.7)	$s - d = - 0.157(d.10^{-3})(h.10^{-3})$
Correction on 100 m	$= - 0.0074$
Scale + Sea-level	$+ 0.0043$
From annexure D	$+ 0.0045$ (Check)

Calculation of temperature difference

Effect of 1° on 100 m $= 0.0011$

Temperature effect equivalent to 0.0043 is $\frac{0.0043}{0.0011} = 3.9°$ C

Reference temperature

The correction is positive, that is, plane distance is greater than the measured distance. Decrease reference temperature below standard temperature. This results in an additional positive correction to measurements, as required.

SST Reference Temperature 22° − 3.9° = 18.1°
= 18° C (rounded off)

6.5 *Grid convergence, γ*

Determinations of direction by astronomical (or gyroscopic) observations are based on the meridian and are azimuths. Grid bearings, β, are obtained from such azimuths, α, by the application of the grid convergence, γ.

Astronomical azimuths will normally be required in the following cases:

1. For isolated traverses connected to the I.S.G. system at one end only, where a check is provided on grid bearing by astronomical observations at the remote end. For example, when provisional co-ordinates are required from a survey and connection is practicable by traverse to one established control point only.

2. For traverses (e.g., road surveys) with a large number of angles where it is not possible to check grid bearings at specified distances along the traverse by other means.

3. If the misclose in bearings transferred from one control point through a traverse to another control point is too large, one method of isolating the error is by means of observed astronomical azimuth.

4. If grid bearings are required for a survey not connected to the Integrated Survey system. Before the extension of State survey control into an area it is often desirable to orient surveys on grid north, as this facilitates the later incorporation of such surveys into the I.S.G. system.

The astronomical determination of azimuth is covered in detail in part 5.

Determining the correction. Grid convergence required for the computation of a grid bearing, may be computed from known I.S.G. co-ordinates by the methods indicated in para. 5.6 (equations (5.3), (5.4)).

In cases 1, 2 and 3 the co-ordinate values required for the formulae in para. 5.6 may be obtained with sufficient accuracy from preliminary computations.

In 4 the values may be obtained graphically from available large scale maps, e.g., parish maps, on which the positions of some survey control points are shown.

The accuracy requirements are indicated by the following:

An error of 1″ in grid convergence is caused by:

(1) an error in the Easting of 40 m, in latitude 38°, at any Easting;

(2) an error in the Easting of 60 m in latitude 28°, at any Easting;

(3) an error in the Northing of 1 400 m, on the zone boundary.

6.6 *Control surveys and high precision surveys*

A higher accuracy will be required for surveys carried out to establish control for survey integration and for high precision engineering or similar surveys.

Projection corrections are always applied in triangulation or control traverse surveys. If such surveys are carried out for the purpose of survey integration, the computation should normally be effected by an adjustment using the method of least squares, using procedures approved by the Surveyor General. If requested the Department of Lands will assist in the computations or will undertake the adjustments. Such adjustments are normally carried out from the data listed on standard forms by the head office of the Department of Lands. This procedure applies to other computations such as resection, intersection, radiation or combination of these techniques.

Projection corrections may be calculated to the precision required using the formulae for δ in para. 5.5; the method involving I.S.G. Tables for the calculation of γ in para. 5.6; and the full formula for K in para. 5.8.

7. CONVERSION OF CO-ORDINATES — GEOGRAPHIC TO GRID AND GRID TO GEOGRAPHIC

7.1 *Introduction*

Integrated Survey Grid co-ordinates may be computed from geographic co-ordinates on the Australian National Spheroid and *vice versa* by the method of conversion described below. I.S.G. Tables are available for manual computation.

The general method for transformation between A.M.G. and I.S.G. co-ordinates is to convert the given co-ordinates to geographicals as intermediate values and thence to the required projection values. In transforming from A.M.G. co-ordinates into I.S.G. values, the operation is achieved in two steps. First A.M.G. values are converted into geographical co-ordinates using the method given in the *Technical Manual of the Australian Map Grid.* Then the geographical co-ordinates are converted to I.S.G. co-ordinates in the appropriate zone, using the method described in this section.

Paragraph 8.1 shows how the operation is simplified in the case of conversions to the central I.S.G. zone. The method of para. 8.3 could also be applied for the conversion.

For the purposes of this section, the symbols ϕ, ϕ', x and m are used to designate the absolute values of these quantities, that is they will all be regarded as positive, even though they are in the Southern Hemisphere.

7.2 *Meridian distance, m,* required for the computation of term a_0 in the conversion formulae has been evaluated from the following formulae based on Jordan Eggert: *Handbuch der Vermessungskunde*, volume III, 1, page 263, 1939 edition.

$$A = 1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 - \frac{5}{256}e^6 - \frac{175}{16\,384}e^8$$

$$B = \frac{3}{8}\left(e^2 + \frac{1}{4}e^4 + \frac{15}{128}e^6 + \frac{35}{512}e^8\right)$$

$$C = \frac{15}{256}\left(e^4 + \frac{3}{4}e^6 + \frac{35}{64}e^8\right)$$

$$D = \frac{35}{3\,072}\left(e^6 + \frac{5}{4}e^8\right)$$

$$m = a(A\,\phi - B\sin 2\,\phi + C\sin 4\,\phi - D\sin 6\,\phi)$$

$$a_0 = 5\,000\,000 - 0.99994\text{ m metres}$$

7.3 *Formulae for Conversion*

Redfearn's formulae, published in *Empire Survey Review* No. 69, 1948, have been adopted for the computation of the tabulated functions in the I.S.G. Tables. The formulae have been arranged so as to give the accuracy specified below even at the edges of the overlaps.

Terms of the 5th and higher order have been omitted with the exception of term E_5, which is represented in a critical table.

Accuracy: nearest 0.001 metre in co-ordinates, 0″.0001 in geographical position and 0″.01 in meridian convergence.

7.4 *Conversion of Co-ordinates: Geographic to Integrated Survey Grid*

Terms $a_0 - a_4$, b_1 and b_3 in the Integrated Survey Grid tables are computed from the formulae:

$$a_0 = 5\,000\,000.000 - 0.99994\,(6\,367\,471.84853\,\phi - 16\,038.95495\sin 2\phi + 16.8335\sin 4\phi - 0.0220\sin 6\phi)$$

$$a_1 = k_0\,\nu\cos\phi\sin 1''.10^4$$

$$a_2 = \frac{k_0}{2}\,\nu\sin\phi\cos\phi\sin^2 1''.10^8$$

$$a_3 = \frac{k_0}{6}\,\nu\cos^3\phi\left(\frac{\nu}{\rho} - t^2\right)\sin^3 1''.10^{12}$$

$$a_4 = \frac{k_0}{24}\,\nu\sin\phi\cos^3\phi\left(4\frac{\nu^2}{\rho^2} + \frac{\nu}{\rho} - t^2\right)\sin^4 1''.10^{16}$$

$$b_1 = \sin\phi.10^4$$

$$b_3 = \frac{\sin\phi}{3}\cos^2\phi\left(2\frac{\nu^2}{\rho^2} - \frac{\nu}{\rho}\right)\sin^2 1''.10^{12}$$

Example. Given: $\phi = 28° 45' 01''.2592$

$\lambda = 148° 15' 02''.1012$

Compute: E, N and γ on Zone 55/2

The computation is carried out on a standard form (annexure E).

The zone number and central meridian are determined from the table on the form. Adopting zone 55/2, the central meridian is 147°.

For interpolating in the I.S.G. Tables, enter the seconds of ϕ on the calculator so that by the multiplication of this quantity with the tabular values for diff. 1″, all increments to the tabular values of the terms for the minutes of the latitude may be calculated. For diff. 1″ take the value tabulated opposite the given minutes, without interpolation.

After terms $a_0 - a_4$, b_1 and b_3 have been entered on the form, compute

$$\omega = \lambda - \lambda_0 \text{ and } p = 0.0001\ \omega''$$

$$y = a_1 p + a_3 p^3$$

$$N = a_0 - a_2 p^2 - a_4 p^4$$

$$\gamma'' = b_1 p + b_3 p^3$$

The formulae will give correct signs if p is entered with the correct sign, i.e. p positive east and negative west of the central meridian. Alternatively terms ω and p can be taken as positive in the formulae for the computation of absolute values of y and γ, and the signs of the answers determined from the following conventions:

East of central meridian, $E = 300\,000 + y$ and γ is positive

West of central meridian, $E = 300\,000 - y$ and γ is negative

7.5 *Conversion of Co-ordinates: Integrated Survey Grid to Geographic*

Geographical (Geodetic) co-ordinates may be directly converted into the co-ordinates of any grid system provided that the same geodetic datum is used for both systems. Conversion formulae are available for all projections and tabulated functions are provided for most of the commonly used systems.

Therefore one method of obtaining corresponding values on another grid system for points on I.S.G. is by means of geographic co-ordinates.

I.S.G. Tables may be used for the conversion of I.S.G. co-ordinates into geographic co-ordinates. The terms $c_1 - c_4$, d_1, d_3 and E_5 in these tables are computed by the following formulae:

$$c_1 = \frac{1}{k_0} \frac{\sec \phi'}{\nu'} \frac{10^6}{\sin 1''}$$

$$c_2 = \frac{1}{2k_0^2} \frac{t'}{\rho'\nu'} \frac{10^{12}}{\sin 1''}$$

$$c_3 = \frac{1}{6k_0^3} \frac{\sec \phi'}{\nu'^3} \left(\frac{\nu'}{\rho'} + 2t'^2 \right) \frac{10^{18}}{\sin 1''}$$

$$c_4 = \frac{1}{24k_0^4} \frac{t'}{\rho'\nu'^3} \left[-4\frac{\nu'^2}{\rho'^2} + 9\frac{\nu'}{\rho'}(1 - t'^2) + 12t'^2 \right] \frac{10^{24}}{\sin 1''}$$

TABLE III

Transformation of Co-ordinates

Geographic to Integrated Survey Grid

Zone: 55/2 Station: P1

Zone:	54/2	54/3	55/1	55/2	55/3	56/1	56/2
C.M.:	141°	143°	145°	147°	149°	151°	153°

Latitude ϕ		28° 45′ 01″.2592	$p = 0.0001\ \omega''$	+	0.45021 012
Longitude λ		148° 15′ 02″.1012	p^2	+	0.20268 915
Central Merid. λ_0		147°	p^3	+	0.09125 3
$\omega = \lambda - \lambda_0$		1° 15′ 02″.1012	p^4	+	0.0411
ω''		4502.1012			
a_0 Tabular value		1 818 618.981	a_1 Tabular value		271 297.039
Increment	−	38.764	Increment	−	0.904
a_0 for ϕ		1 818 580.217	a_1 for ϕ		271 296.135
a_2 Tabular value		3 163.187	a_3 Tabular value		57.526
Increment	+	0.025	Increment	−	0.001
a_2 for ϕ		3 163.212	a_3 for ϕ		57.525
a_4 for ϕ		2.260	$a_1\ p$	+	122 140.266
a_0	+	1 818 580.217	$a_3\ p^3$	+	5.249
$a_2\ p^2$	−	641.149	y		122 145.515
$a_4\ p^4$		0.093	False Origin		300 000.000
NORTHING N		1 817 938.975	EASTING E		422 145.515
b_1 Tabular value		4 809.888			
Increment		0.054			
b_1 for ϕ		4 809.942			
$b_3\ \phi\lambda$		2.94			
$b_1 \cdot p$	+	2165.485			
$b_3 \cdot p^3$	+	0.268			
$\pm\gamma$	+	2165.753			
GRID CONVERGENCE γ	+	0° 36′ 05″.753			

Formulae

$$y = a_1\ p + a_3\ p^3$$

$$E = 300\ 000 + y$$

$$N = a_0 - a_2\ p^2 - a_4\ p^4$$

$$\gamma = b_1\ p + b_3\ p^3$$

Answers: E 422 145.551 N 1 817 938.975

γ + 36′ 05″.75

$$E_5 = \sec\phi' \frac{y^5}{120k_0^5\nu'^5}\left[-4\frac{\nu'^3}{\rho'^3}(1-6t'^2)+\frac{\nu'^2}{\rho'^2}(9-68t'^2)\right.$$
$$\left.+72\frac{\nu'}{\rho'}t'^2+24t'^4\right]$$

$$d_1 = \frac{1}{k_0}\frac{t'}{\nu'}\frac{10^6}{\sin 1''}$$

$$d_3 = \frac{1}{3k_0^3}\frac{t'}{\nu'^3}\left[-2\frac{\nu'^2}{\rho'^2}+3\frac{\nu'}{\rho'}+t'^2\right]\frac{10^{18}}{\sin 1''}$$

Example. Given: $E = 422\,145.515$, $N = 1\,817\,938.975$ } zone 55/2

Compute: ϕ, λ, γ

The computation is carried out on a standard form (annexure F).

The central meridian is determined from the table on the form. For zone 55/2 the value is 147°

$y = E - 300\,000$ $\qquad q = 0.00000\,1\,y$

To obtain foot point latitude, ϕ', interpolate the northing in the tabular values of N (pages 35-43 of the tables).

Thus $1\,818\,000 - 1\,817\,938.975 = 61\,025$

$\phi' = 28°45'20''.1067 + 0.06102\,5 \times 32.48348 = 28°45'22''.0890$

The $\Delta\phi'$ value of 32″.48348 is taken without interpolation as shown opposite the tabulated value of N which is the closest to and greater than the given Northing.

Alternatively ϕ' may be obtained by an inverse interpolation in the tabular values of the term a_0 as follows: select the term a_0 the value of which is closest to and greater than the given Northing, note the degrees and minutes of the corresponding latitude and diff. 1″, without interpolation, on the line containing the term. Divide the difference $(a_0 - N)$ by diff. 1″ to obtain the required increment in seconds.

Compute increments to tabular values of terms where required as explained in para. 7.4 and note all terms on the form. Then using the formulae below compute:

$$\phi = \phi' - c_2q^2 + c_4q^4$$
$$\omega = c_1q - c_3q^3 + E_5$$
$$\gamma = d_1q - d_3q^3$$

E_5 is obtained from a critical table. The maximum value of E_5 is +0″.0002 at the edge of zone overlaps. It may be omitted within the zone boundaries.

The formulae will give correct signs if q is entered with its appropriate sign, positive east and negative west of the central meridian. Alternatively compute the absolute values of ω and γ taking the sign of q and all tabulated terms in the formulae as positive, and follow the sign conventions:

If y is negative ($E < 300\,000$), $\lambda = \lambda_0 - \omega$ and γ is negative

If y is positive ($E > 300\,000$), $\lambda = \lambda_0 + \omega$ and γ is positive

7.6 *Calculation of foot point latitude from meridian arc*

The foot point latitude, ϕ', is required for the conversion of co-ordinates from grid to geographic. It may be obtained by an inverse interpolation between the tabular values of a_0 in the I.S.G. Tables (*see* para. 7.5). However, direct interpolation may be more convenient especially for the preparation of computer programmes since it avoids reiteration.

TABLE IV

Transformation of Co-ordinates

Integrated Survey Grid to Geographic

Zone: 55/2 Station: P1

Zone:	54/2	54/3	55/1	55/2	55/3	56/1	56/2
C.M.:	141°	143°	145°	147°	149°	151°	153°

EASTING E:		422 145.515	NORTHING N:		1 817 938.975
False Origin		300 000.000	$q = 0.00000\,1y$	+	0.12214 5515
$y \pm$		122 145.515	q^2	+	0.01491 9527
c_1 Tabular value		36 859.967	q^3	+	0.00182 235
Increment	+	2.155	q^4	+	0.00022 26
c_1		36 862.122			
c_3 Tabular value	+	242.352	Interp. N ϕ'		28° 45′ 22″.0890
Increment	+	0.060	c_2 Tabular value		1 396.040
c_3		242.412	Increment	+	0.353
$c_1\,q$	+	4 502″.5429	c_2		1 396.393
$c_3\,q^3$	−	0″.4418			
E_5	+	0″.0001	c_4		16.8
			ϕ'		28° 45′ 22″.0890
$\pm\,\omega$	+	1° 15′ 02″.1012	$c_2\,q^2$	−	20″.8335
Central Merid. λ_0		147°	$c_4\,q^4$	+	0″.0037
LONGITUDE λ		148° 15′ 02″.1012	LATITUDE ϕ		28° 45′ 01″.2592
d_1 Tabular value		17 729.23			
Increment		4.497	Formulae:		
d_1		17 733.727	$\phi = \phi' - c_2\,q^2 + c_4\,q^4$		
d_3		188.1	$\omega = c_1\,q - c_3\,q^3 + E_5$		
$d_1\,q$	+	2 166″.095	$\gamma = d_1\,q - d_3\,q^3$		
$d_3\,q^3$	−	0″.343			
γ''	+	2 165″.752	$\lambda = \lambda_0 \pm \omega$		
GRID CONVERG. γ	+	0° 36′ 05″.752			

Answers: ϕ 28° 45′ 01″.2592

λ 148° 15′ 02″.1012

γ + 36′ 05″.76

The following formulae, where all angular quantities are in radians, may be used for the computation of ϕ' from the corresponding meridian arc x.

$$n = \frac{f}{2-f} = 1/\left(\frac{2}{f} - 1\right) = \frac{1}{595.50} \text{ exactly (on the Australian National Spheroid)}$$

$$G = \frac{a}{1+n}\left(1 + \frac{1}{4}n^2 + \frac{1}{64}n^4 + \ldots\right)$$

$$\varepsilon = \frac{x}{G}$$

$$\phi' = \varepsilon + \frac{3}{2}\left(n - \frac{9}{16}n^3\right)\sin 2\varepsilon + \frac{21}{16}n^2 \sin 4\varepsilon + \frac{151}{96}n^3 \sin 6\varepsilon$$

On the Integrated Survey Grid:

$$x = 5\,000\,000 - N$$

$$n = 0.00167\,92611\,25105$$

$$\varepsilon = \frac{x}{6\,367\,089.8002} \text{ (including factor } k_0 = 0.99994)$$

$$\phi' = \varepsilon + 0.00251\,88876\,92 \sin 2\,\varepsilon + 0.00000\,37011\,423 \sin 4\,\varepsilon + 0.00000\,00074\,4836 \sin 6\,\varepsilon$$

The values of ϕ' at 1 000 metre intervals have been tabulated in the I.S.G. Tables.

7.7 *Zone to Zone Transformation*

In the vicinity of zone boundaries, the co-ordinates of a point will frequently be required referred to the origin of both of the neighbouring zones.

One method of computation of co-ordinates from zone to zone is to convert the given grid co-ordinates into geographic co-ordinates and then, using the central meridian of the adjacent zone, convert the geographic co-ordinates into grid co-ordinates.

If geographic co-ordinates are not required, a direct solution is available. The method proposed by E. Gotthardt and published in a German textbook by W. Grossman: *Geodetische Rechnungen und Abbildungen*, 1964 edition, page 188, has been adopted in this *Manual*. The mathematical theory of the method is also discussed by R. A. Hirvonen and W. Hristow in the periodical: *Zeitschrift fur Vermessungswesen*, 1938, pages 321 and 534.

The formulae for the coefficients k_1–k_6 are quoted below. Values of these coefficients are tabulated in annexure G for the Integrated Survey Grid, New South Wales, for latitudes 28°–38° *S*.

$$k_1 = +\frac{1}{3N_0^2}\cos^2\phi_0\,(3 - 4\tan^2\phi_0)\,\omega_0^2$$

$$k_2 = -\frac{1}{3N_0^2}\sin\phi_0\,(1 + 5\tan^2\phi_0)\,\omega_0 - \frac{1}{9N_0^2}\sin\phi_0\cos^2\phi_0\,(37 - 26\tan^2\phi_0)\,\omega_0^3$$

$$k_3 = -\frac{1}{N_0}\cos\phi_0\,(1 + e'^2\cos^2\phi_0)\,\omega_0 + \frac{1}{6N_0}\cos^3\phi_0\,(1 + 31\tan^2\phi_0)\,\omega_0{}^3$$

$$k_4 = +\frac{3}{N_0}\sin\phi_0\cos\phi_0\,(1 + e'^2\cos^2\phi_0)\omega_0{}^2$$

$$+\frac{1}{2N_0}\sin\phi_0\cos^3\phi_0\,(1 - 13\tan^2\phi_0)\omega_0{}^4$$

$$k_5 = -2\sin^2\phi_0\,\omega_0{}^2 - \frac{2}{3}\sin^2\phi_0\cos^2\phi_0\,(2 - \tan^2\phi_0)\,\omega_0{}^4$$

$$k_6 = -2\sin\phi_0\omega_0 - \frac{2}{3}\sin\phi_0\cos^2\phi_0\,(1 - 2\tan^2\phi_0 + 3e'^2\cos^2\phi_0)\omega_0{}^3$$

$$-\frac{2}{15}\sin\phi_0\cos^4\phi_0\,(2 - 11\tan^2\phi_0 + 2\tan^4\phi_0)\,\omega_0{}^5$$

Values of k_1-k_6 are calculated for points at 30-minute intervals along the zone boundary. ϕ_0 is the latitude of the point selected which is closest to the latitude of the point being transformed.

ω_0 is half of the zone width, in radians.

Assuming that

$$E - E_0 = \Delta E \text{ and } E.10^{-5} = (y)$$
$$N - N_0 = \Delta N \text{ and } N.10^{-5} = (x)$$

where E_0 and N_0 are I.S.G. co-ordinates for the adopted point on the zone boundary with latitude ϕ_0. New co-efficients K_3 to K_6 are calculated as follows:

$$k_1(y) - k_2(x)k_3 = K_3 \qquad k_1(x) + k_2(y)k_4 = K_4$$
$$K_3(y) - K_4(x)k_5 = K_5 \qquad K_3(x) + K_4(y)k_6 = K_6$$

The transformed co-ordinates are then computed by the formulae:

$$E' = E'_0 + \Delta E + K_5(y) - K_6(x)$$
$$N' = N'_0 + \Delta N + K_5(x) + K_6(y)$$

Symbols and sign convention:

West zone to east zone: $E_0 = E'_{0\text{ west}}$; $E'_0 = E'_{0\text{ east}}$
k_2, k_6 positive and k_3 negative

East zone to west zone: $E_0 = E_{0\text{ east}}$; $E'_0 = E'_{0\text{ west}}$
k_2, k_6 negative and k_3 positive

Note: $E'_{0\text{ west}}$ and $E'_{0\text{ east}}$ are values tabulated in annexure G.

Method of Calculation. Select a line in the table for which the tabulated Northing N_0 is the closest to the given N value. Enter the values shown on the selected line, *without any interpolation,* for co-efficients k_3 to k_6 on the Form in annexure H, with the corresponding signs.

Determine the signs for those terms shown as absolute values only, according to the rules shown at the bottom of the table.

Enter also values for E, N, E'_0, N'_0. Compute and enter ΔE, ΔN, (y), (x). Arithmetic operations are carried out in the order indicated on the form.

Check the result by the transformation of the obtained values back to the originally given values, using lower half of the form.

Example.

Given: Co-ordinates of Point A (metres) in West zone:

$E =$ 422 145.515 Zone: 55/2

$N =$ 1 817 938.975

Calculate: The corresponding values of Point A in the East Zone

Selecting a position on zone boundary at latitude of 28°30′ in the table of annexure G the computations are carried out on annexed form H.

TABLE V

Zone to Zone Transformation

Zone	54/2	54/3	55/1	55/2	55/3	56/1	56/2
C.M.	141°	143°	145°	147°	149°	151°	153°

Station A Zone 55/2 to Zone 55/3

E	422 145.515	N	1 817 938.975
E_0	397 901.321	N_0	1 845 917.131
ΔE	+ 24 244.194	ΔN	− 27 978.156
(y)	+ 0.24244 2	(x)	− 0.27978 2
$k_1(y)$	+ 0.0008	$k_1(x)$	− 0.0010
$-k_2(x)$	+ 0.0196	$k_2(y)$	+ 0.0170
k_3	− 24.1467	k_4	− 0.6035
K_3	− 24.1263	K_4	− 0.5875
$K_3(y)$	− 5.8492	$K_3(x)$	+ 6.7501
$-K_4(x)$	− 0.1644	$K_4(y)$	− 0.1424
k_5	− 13.8729	k_6	+ 1 665.6539
K_5	− 19.8865	K_6	+ 1 672.2616
$K_5(y)$	− 4.8213	$K_5(x)$	+ 5.5639
$-K_6(x)$	+ 467.8687	$K_6(y)$	+ 405.4264
ΔE	24 244.194	ΔN	− 27 978.156
E'_0	202 098.679	N'_0	1 845 917.131
E'	226 805.920	N'	1 818 349.965

Answer: E 226 805.920 Zone 55/3

N 1 818 349.965

8. TRANSFORMATION OF CO-ORDINATES FROM ONE CO-ORDINATE SYSTEM INTO ANOTHER

8.1 *Transformation from Australian Map Grid to Integrated Survey Grid co-ordinates*

The transformation of co-ordinates from the Australian Map Grid system into the Integrated Survey Grid system and *vice versa* can be carried out directly through geographical values as noted in para. 7.1.

Each 6° A.M.G. zone is covered by three 2° I.S.G. zones. In the middle zone of these three I.S.G. zones, the conversion is simple because both projections have the same central meridian. Only three steps are necessary for conversion from A.M.G. co-ordinates (suffix $_6$) to I.S.G. (suffix $_2$)

(1) Transform A.M.G. co-ordinates from false origin to true origin

$$y_6 = E_6 - 500\,000 \qquad x_6 = N_6 - 10\,000\,000$$

(2) Multiply by ratio of central scale factors

$$\frac{0.99994}{0.9996} = 1.00034\,0136$$

(3) Transform I.S.G. co-ordinates from true origin to false origin

$$E_2 = y_2 + 300\,000 \qquad N_2 = x_2 + 5\,000\,000$$

Example. Conversion from A.M.G. 55 to I.S.G. 55/2 co-ordinates

On A.M.G. 55 $\quad E$ 622 103.983 $\quad N$ 6 819 020.940

$$\begin{aligned} y_2 &= (E_6 - 500\,000) \times 1.00034\,0136 \\ &= (622\,103.983 - 500\,000) \times 1.00034\,0136 \\ &= 122\,145.515 \\ x_2 &= (N_6 - 10\,000\,000) \times 1.00034\,0136 \\ &= (6\,819\,020.940 - 10\,000\,000) \times 1.00034\,0136 \\ &= -3\,182\,061.025 \end{aligned}$$

Thus co-ordinates on I.S.G. 55/2 are:

E 422 145.515 $\quad N$ 1 817 938.975

For the conversion in the converse direction, I.S.G. to A.M.G., the ratio in Step (2) becomes

$$\frac{0.9996}{0.99994} = 0.99965\,9980$$

8.2 *Transformation by plane bearing and distance*

The following method may be used for the transformation of co-ordinates of any co-ordinate system into the I.S.G. system, if co-ordinates of one or more points are known in both systems. The method is suitable for desk top calculators of the programmable or non-programmable type.

Case 1 is the case where it is possible to convert distances and bearings on each of the co-ordinate systems to the corresponding spheroidal distances and azimuths. Such co-ordinate systems include I.S.G., A.M.G. systems and others. Only one point common to both systems is required, but it is preferable to use two such points and to repeat the procedure using a second point, to provide an independent check on the transformed co-ordinates.

The principle of the computation is to convert the distance and bearing between the point whose co-ordinates are known on both systems and the point whose co-ordinates are to be transformed, to the corresponding spheroidal quantities and then work from the spheroid onto the plane of the new (I.S.G.) system

Given: Co-ordinates of A on System 1 and on the I.S.G.

Co-ordinates of X on System 1.

Calculate: Co-ordinates of X on the I.S.G.

Method: (1) Calculate bearing and distance AX on co-ordinate system 1. (Join calculation, *see* para. 10.1.)

(2) Apply arc-to-chord and grid convergence to the bearing, and scale, and, if appropriate, sea level correction to the distance, to obtain the azimuth and spheroidal distance of the corresponding line on the spheroid.

(3) Now calculate arc-to-chord, grid convergence and scale corrections on the I.S.G. system, for the line AX. If this is a very long line it may be necessary to calculate approximate co-ordinates of X on the I.S.G. in order to calculate the corrections to sufficient accuracy. These corrections are then applied to the spheroidal quantities to obtain the plane bearing and distance on the I.S.G. Some care is necessary in determining the signs of the corrections, since the conversion is from spheroid to projection in this case.

(4) Calculate the co-ordinates of X on the I.S.G. from the co-ordinates of A and the bearing and distance AX (radiation calculation, *see* para. 10.2).

(5) As a check, repeat the transformation using a second Point B whose co-ordinates are known on both systems.

Case 2. It is not always possible to convert data on a co-ordinate system to the corresponding values on the spheroid. The formulae relating them may not be known, or the data for the conversion may not be available. Case 2 deals with this situation. It is still possible to carry out the transformation if there are at least two points whose co-ordinates are known on both systems. Two such points is the minimum, but it is preferable to use a third in order to provide a check on the results. The point for transformation should be within the triangle formed by the three points, for example in figure 4, X is within the figure ABC, but Y is not. Points AB and D should be used for its transformation. To reduce the effects of changes in scale and orientation in the survey, the distances between A,B,C and X should not be large.

The principle of the computation is to use distances and bearings of a line AB common to both systems, to determine the scale and orientation differences between them. These differences are then applied to the lines AX and BX to calculate X on the second system.

Given: Co-ordinates of A and B on Systems 1 and 2.
Co-ordinates of X on System 1.

Calculate: Co-ordinates of X on System 2.

Method: (1) Calculate bearings and distances AB on both systems.

(2) Deduce the transformation scale factor, which is the length AB on System 2 over the length on System 1; and the transformation orientation correction which is the bearing of AB on System 2 minus the bearing on System 1.

(3) Calculate the bearings and distances AX and BX on System 1.

(4) Apply the transformation scale factor and orienting correction to obtain AX and BX on System 2.

(5) Calculate co-ordinates of X on System 2 from A and, as a check on its computation only, from B.

(6) As an independent check, repeat the computation using a third point, C, in place of B. This will generally not produce identical results for X, but they should be sufficiently close that the mean can be adopted.

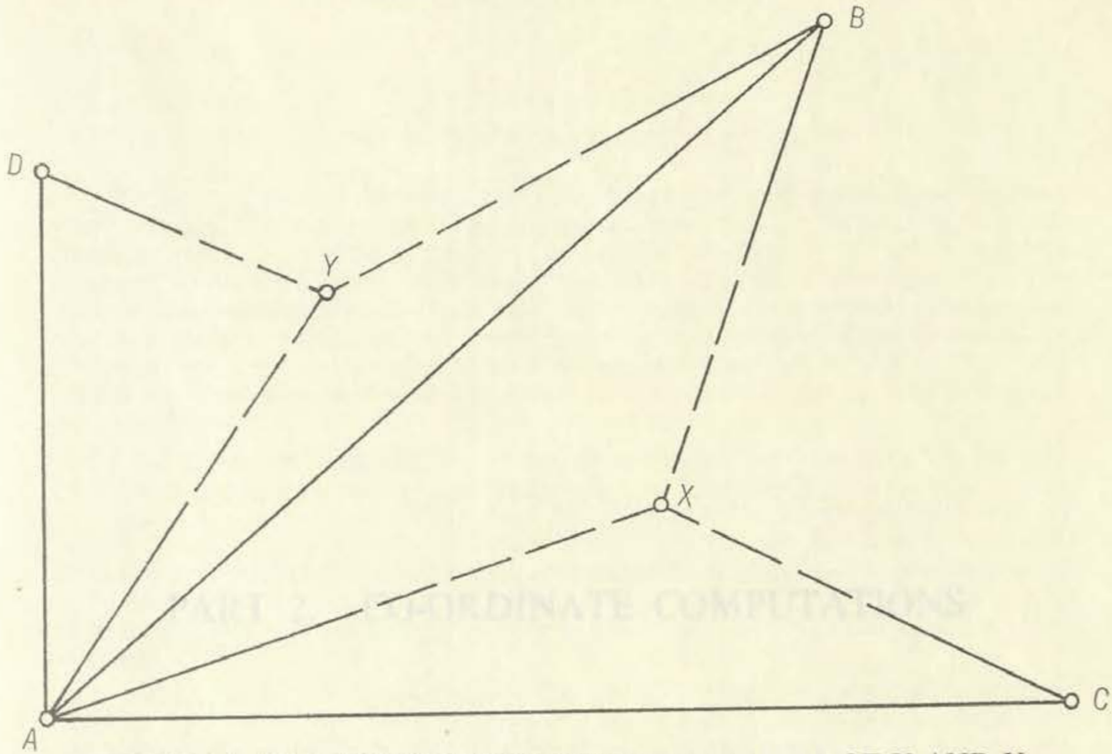


FIGURE 4—TRANSFORMATION OF CO-ORDINATES OF X AND Y

8.3 *Lauf's transformation*

The transformation from any conformal co-ordinate system into another may be effected by the method of Professor Lauf. Complex number arithmetic is used for the transformation and the procedure is based on co-ordinates known in both systems for three or four common points. The points to be transformed should be situated inside the area formed by the lines joining these points.

This is a more sophisticated procedure than that of the previous paragraph, because it takes into account variations in scale and orientation over the area covered by the common points. Normally it would only be attempted using an electronic computer, and programmes are available for transformation by this method.

To detect possible errors in the co-ordinates of these points and to provide a check on the general accuracy, other points, the co-ordinates of which are known in both systems, may be included as points to be transformed.

This method is suitable for the transformation of co-ordinates from the yard grid system on the Clarke's 1858 spheroid into the I.S.G. system. However, care should be taken to maintain consistency in co-ordinates. The yard values of the common points used as basis for the transformation should correspond with the values used for co-ordination of the points to be transformed.

The method may also be used for the transformation of co-ordinates of a local system into the I.S.G. system. In local systems, generally no projection corrections have been applied and such systems may be assumed to be conformal for the purpose, if the extent of the area covered in E-W direction is reasonably small.

For the transformation of co-ordinates of the Cassini-Soldner system, the terms $(y^3/6R^2 + y^5/24R^4)$ should be added to the Cassini-Soldner Y co-ordinate (departure) to obtain corresponding conformal values, before commencing the transformation computation.

PART 2. CO-ORDINATE COMPUTATIONS

9. CO-ORDINATE COMPUTATIONS

Part 2 of the *Manual* deals with co-ordinate computations on the projection plane. Section 10 indicates, for the basic computations, the data, the purpose and the formulae used. It provides a standard layout and an example of each and draws attention to special features, such as the method of checking the computation. The main section comprises an example of a control survey, computed by semi-graphic methods, and the connection of a property survey to the control. This is preceded by a section which gives detailed explanations and comments on the calculations, and which, for those who are unfamiliar with this form of computation, should make it easier to follow.

Although the increasing use of pocket and desk-top calculators has brought changes in computations, the forms in this part are basically those of the earlier era of hand calculator and natural tables. This is because the computations, when set out in this fuller format, provide better illustrations and are more easily followed. This format can be readily adapted and abbreviated for electronic calculators, bearing in mind that the process of abbreviation can be overdone. It is valuable to record appropriate intermediate results, to help in checking and to provide data for other purposes.

The example is calculated to a higher precision than is justified by the circumstances. Again this is in order to provide a better illustration of the computations, the corrections and the checks. This example is also atypical of surveys on the I.S.G. in other respects. Normally, far more use would be made of edm measurements and a smaller proportion of the survey would be devoted to extending the control into the property survey. This is discussed in more detail in section 13.

10. STANDARD PLANE CO-ORDINATE COMPUTATIONS

10.1 *The Join*

In the *join* or *bearing and distance* calculation, the co-ordinates of two points A and B are given and it is required to compute the plane bearing and distance.

Formulae

$$\tan\theta = \frac{E_B - E_A}{N_B - N_A} = \frac{\Delta E}{\Delta N} \text{ or} \qquad \ldots(10.1)$$

$$\cot\theta = \frac{N_B - N_A}{E_B - E_A} = \frac{\Delta N}{\Delta E} \qquad \ldots(10.2)$$

$$S = \frac{\Delta E}{\sin\theta} \qquad \ldots(10.3)$$

$$S = \frac{\Delta N}{\cos\theta} \qquad \ldots(10.4)$$

Standard Form

A	E_A	N_A	$\cot\theta$ or $\tan\theta$	θ
B	E_B	N_B	$\sin\theta$	S
	$E_B - E_A$	$N_B - N_A$	$\cos\theta$	S

Comments

1. If ΔE is smaller than ΔN, use equations (10.1) and (10.4) to determine θ and S, as they will yield the more accurate values. If ΔN is the smaller, use (10.2) and (10.3).

2. The second value of S is calculated as a check on the computation. Note however that it is not a complete check; it is not capable of detecting errors in the determination of ΔE or ΔN, or in looking up or writing out the bearing θ. These need to be checked independently.

3. The quadrant of θ is determined from the signs of ΔE and ΔN. If θ' is the first quadrant angle determined from equation (10.1) or (10.2), then θ is deduced as shown in the final column of Table VI.

TABLE VI

	Quadrant	Sign of ΔE	Sign of ΔN	Calculation of θ
0 — 90°	First	+	+	$\theta = \theta'$
90 — 180°	Second	+	—	$\theta = 180° - \theta'$
180 — 270°	Third	—	—	$\theta = 180° + \theta'$
270 — 360°	Fourth	—	+	$\theta = 360° - \theta'$

Example

A	422 145.515	1 817 938.975	cot 0.41909 849	292°44′18″.54
B	398 112.145	1 828 011.324	— 0.92227 868	26 058.685
	—24 033.370	+ 10 072.349	+ 0.38652 558	26 058.687

Notes

1. $\Delta N < \Delta E$, therefore use equation (10.2), *cotangent* formula for θ, and equation (10.3) *sine* formula for S.

2. $\theta' = 67°15'41''.46$. Signs of ΔE, ΔN are —, +. Thus θ is in fourth quadrant, $\theta = 360° - \theta'$.

3. In the *Numerical Example*, section 12, the join is used for orientation on page *3* and for checking directions on pages *15* and *16*.

10.2 *The Radiation*

The *radiation* or *polar calculation* is the converse of the join. The co-ordinates of point A and the bearing and distance to point B are given and it is required to calculate co-ordinates of B.

Formulae

$$E_B = E_A + S \sin\theta \qquad \ldots(10.5)$$

$$N_B = N_A + S \cos\theta \qquad \ldots(10.6)$$

Standard Form

A			E_A	N_A
	S	$\sin\theta$	$S\sin\theta$	$S\cos\theta$
	θ	$\cos\theta$		
B			E_B	N_B

Comments

1. Correct signs of $\sin\theta$ and $\cos\theta$ must be applied. These signs are the same as those of ΔE and ΔN in table VI, para 10.1.

2. The computation is unchecked. Usually a radiation is checked in the field by independent measurements. These are calculated to yield independent co-ordinates for the point, thus checking field work and calculations. Alternatively the calculation may be checked using:

$$\Delta E + \Delta N = \sqrt{2}\, S \sin(\theta + 45°) \qquad \ldots(10.7)$$

$$\Delta E - \Delta N = -\sqrt{2}\, S \cos(\theta + 45°) \qquad \ldots(10.8)$$

Example

A			422 145.515	1 817 938.975
	26 058.685	−0.92227 868	−24 033.370	+ 10 072.348
	292°44′18″.54	+0.38652 558		
B			398 112.145	1 828 011.323

Note Examples of the radiation calculation in the *Numerical Example*, section 12 are on page *7* where it is used to obtain the preliminary co-ordinate, and on pages *15* and *16* where it is used for fixing and checking co-ordinates of corners.

10.3 *The Intersection*

In the *intersection* or *triangle* calculation, the co-ordinates of two points *A* and *B* are given and the bearings θ_1, θ_2 from *A* and *B* to a third point *P*. The co-ordinates of *P* are required.

Formulae

$$N_P = N_A + \frac{(N_B - N_A)\tan\theta_2 - (E_B - E_A)}{\tan\theta_2 - \tan\theta_1} \qquad \ldots(10.9)$$

$$N_P = N_B + \frac{(N_B - N_A)\tan\theta_1 - (E_B - E_A)}{\tan\theta_2 - \tan\theta_1} \qquad \ldots(10.10)$$

$$E_P = E_A + (N_P - N_A)\tan\theta_1 \qquad \ldots(10.11)$$

$$E_P = E_B + (N_P - N_B)\tan\theta_2 \qquad \ldots(10.12)$$

$$S_1 = (N_P - N_A)\sec\theta_1 = (E_P - E_A)\operatorname{cosec}\theta_1$$

$$S_2 = (N_P - N_B)\sec\theta_2 = (E_P - E_B)\operatorname{cosec}\theta_2$$

Standard Form

A	E_A	N_A	θ_1	
B	E_B	N_B	θ_2	
	$E_B - E_A$	$N_B - N_A$		
$\sec\theta_1$	$E_P - E_A$	$N_P - N_A$	$\tan\theta_1$	S_1
$\sec\theta_2$	$E_P - E_B$	$N_P - N_B$	$\tan\theta_2$	S_2
			$(\tan\theta_2 - \tan\theta_1)$	
P	E_P	N_P		

Comments

1. N_P and E_P are each calculated twice so as to provide a check on the computation. This is a rigorous check, except that an error in determining $(\tan\theta_2 - \tan\theta_1)$ will not be detected. This subtraction needs to be checked independently.

2. Correct signs must be applied. Tan θ is $+$ in first and third quadrants, $-$ in second and fourth.

Example

A	422 145.515	1 817 938.975	237°14′21″.6	
B	398 112.145	1 828 011.324	165 53 42.8	
	−24 033.370	+ 10 072.349		
−1.84798 12	−18 509.664	− 11 910.688	+1.55403 82	22 010.73
−1.03108 55	5 523.706	− 21 983.037	−0.25127 13	22 666.39
			−1.80530 95	
P	403 635.851	1 806 028.287		

Note The intersection calculation is used on pages *8* and *9* of the *Numerical Example* to calculate preliminary co-ordinates of points.

10.4 *The Cut*

Given a bearing θ from a known point C, the point where the bearing cuts any specified co-ordinate value can be determined. In particular, the "cut" of the bearing on the preliminary co-ordinate of a point P is calculated in order to portray graphically the rays used in fixing P.

Given preliminary co-ordinates of P: $E_{(P)}$, $N_{(P)}$. Required to determine where the ray of bearing θ from C (E_C, N_C) cuts the ordinate $E_{(P)}$ or $N_{(P)}$. The ray cuts $E_{(P)}$ at the point of northing $N_{P'}$; or $N_{(P)}$ at a point of Easting $E_{P'}$.

Formulae

Form (A)

$$E_{P'} = E_C + (N_{(P)} - N_C) \tan \theta \qquad \ldots(10.13)$$

$$S = (N_{(P)} - N_C) \sec \theta \qquad \ldots(10.14$$

or

Form (B)

$$N_{P'} = N_C + (E_{(P)} - E_C) \cot \theta \qquad \ldots(10.15)$$

$$S = (E_{(P)} - E_C) \operatorname{cosec} \theta \qquad \ldots(10.16)$$

Standard Form

	Form (A)	
C	E_C	N_C
		$N_{(P)}$
	$\tan \theta$	$N_{(P)} - N_C$
	$(N_{(P)} - N_C) \tan \theta$	$\sec \theta$
	$E_{P'}$	S
	Form (B)	
C	E_C	N_C
	$E_{(P)}$	
	$E_{(P)} - E_C$	$\cot \theta$
	$\operatorname{cosec} \theta$	$(E_{(P)} - E_C) \cot \theta$
	S	$N_{P'}$

Comments

1. The cut is chosen so that it intersects the ordinate at an angle closer to 90° i.e. greater than 45°. This means that equations (10.13) and (10.14) and Standard Form (A) are used when θ is closer to the north-south direction (315° – 45°; 135° – 225°). In this case ΔN is greater than ΔE and tan θ is less than unity.

Conversely Form (*B*) is used for θ closer to the east-west direction (45° — 135°; 225° — 315°), when *E* is greater than ΔN and cot θ is less than unity.

Example

		Form (*A*)	
C	391 050.617	1 831 727.990	333°54′29″.5
		1 806 028.287	
	−0.48971 76	−25 699.703	
	+12 585.597	1.11347 25	
P′	403 636.214	28 615.912	
		Form (*B*)	
D	387 602.396	1 802 421.729	77 19 21.0
	403 635.851		
	+16 033.455	+0.22494 71	
	1.02498 84	+3 606.679	
P′	16 434.105	1 806 028.408	

Note

1. Signs of ΔE and ΔN are derived from co-ordinates and tan θ or cot θ. As a check, note that they should be consistent with the signs in table VI, para 10.1.

2. The calculation of the cut is used in the *Numerical Example* on pages *7*, *8* and *10** for calculating the positions of fixing rays to be used in the error figures.

10.5 *The Semi-Graphic Intersection*

In the *semi-graphic intersection* the directions observed to fix a point are calculated and then plotted in graphical form. A preliminary position is first determined using the intersection calculation, or, if a distance has been measured to the point, by the radiation calculation. The bearings to or from other points are then used to calculate the cuts on these preliminary co-ordinates. These calculations provide the data for the graph. The semi-graphic intersection is a combination of calculations which have been covered in earlier paragraphs.

The graph shows, in general, that the rays do not all pass through the same point but intersect to form an *error figure* which is triangular if there are three rays, and more complex if there are more. The final point must be chosen so as to minimize the errors. The relative weighting is taken into account if the position of the final point is chosen within the error figure, placed so that its distance from each ray is proportional to the length of the ray, that is, the distance to the other station. When there are three rays this is a simple estimation, but with four or more, inconsistencies may arise and the choice may become a matter of opinion.

Examples of the graphs are shown on pages *7* and *8* of the *Numerical Example*. On page *7* there are five rays and the final point can be chosen by resolving in two directions: the direction of the rays *P*1 and *P*5, and the direction more or less at right angles, being the combination of the rays *P*3, *P*4 and the distance from *P*1.

On page *8* the error figure is triangular, the rays are all observed in the forward direction only and all are of nearly the same length. The chosen point is then simply the centre of gravity of the triangle.

The check is provided by noting that the error figure is small, and that the discrepancies between the plotted rays and the final point are allowable errors of

*The *Numerical Example* is given in section 12 and page numbers in the example are referred to in Italic numerals.

observation. As an independent check, joins can be calculated to all fixing stations, and a comparison made of the differences between observed and calculated bearings. This provides a check on the complete numerical computation, and also an indication whether an appropriate position has been chosen in the error figure.

10.6 *The Resection*

In a *resection* the position of an unknown point is calculated using directions observed from the point to stations of known co-ordinates. Generally three such observations fix the position uniquely, but provide no check. A minimum of four, but preferably five or six suitably positioned stations should be observed to provide checked co-ordinates.

Given observed directions from P to five stations A, B, C, D and E, it is required to calculate co-ordinates of P. Co-ordinates of the five stations are known. In the calculation of the resection proper, three of the rays, say A, B and C, are chosen to calculate a unique solution. They should preferably be the rays providing the strongest fix. The first step is to solve for the orientation. The calculation shown illustrates only one of many methods available. Once the bearing of one of the rays has been calculated all the observed directions can be oriented and the calculation becomes similar to the semi-graphical intersection calculation in para 10.5. The steps which follow are calculations of the preliminary co-ordinates from two rays A and C using the *intersection* calculation, followed by the *cut* of the third ray B. Unless there are errors in the calculation, this cut will pass exactly through the preliminary point. This provides a check on the calculation but not on the observations or data. It is necessary to calculate the cuts from D and E to provide a check.

Formulae

1. Calculate the orientation. See figure 9, page 9 of numerical example. Observations to A, B and C. Angles calculated from observed directions: α subtended by A and B; and β subtended by B and C.

$$\cot \theta_{PB} = \frac{N_A \cot \alpha - N_B(\cot \alpha + \cot \beta) + N_C \cot \beta + (E_C - E_A)}{E_A \cot \alpha - E_B(\cot \alpha + \cot \beta) + E_C \cot \beta - (N_C - N_A)} \quad \ldots(10.17)$$

2. Deduce the orienting swing and apply it to all observed directions.
3. Calculate the preliminary co-ordinates the Intersection of P from A and C.
4. Calculate the cut from B. This cut must pass through the preliminary point.
5. Calculate cuts from D and E.
6. Plot the graph and select the position of the final point.

Standard Form

A	E_A	N_A	$\alpha = \varepsilon_B - \varepsilon_A$	$\cot \alpha$
B	E_B	N_B	$\beta = \varepsilon_C - \varepsilon_B$	$\cot \beta$
C	E_C	N_C		
				$\cot \alpha + \cot \beta$
	$E_C - E_A$	$N_A - N_C$		
	$\cot \theta_{PB}$			
	θ_{PB}			
	ε_B			
	$\varepsilon_B - \theta_{PB}$	= Orienting swing		

This is followed by the calculation of the intersection, the calculation of cuts and the plotting of the graph.

Comment: The danger circle

The resection is a useful method where, in a survey, it is necessary to bring control in from surrounding control points. The advantage is that it requires observations from only one point and this advantage is of greater importance if access to the control points is difficult. However there are some disadvantages: it may be difficult to find a point from which five or six suitably situated control points are visible; and the calculation is somewhat longer and more complex than the standard intersection. The main disadvantage lies in the existence of the *danger circle*.

The circle passing through the fixed points A, B, and C which are observed from P for the resection, is significant. At all points on that circle, the same angle α is subtended between A and B, and the same angle β between B and C. This means that if P is on the danger circle ABC, its position is indeterminate, and if it is close to the danger circle its position is uncertain, or subject to considerable error. The rays to further points D and E may overcome this ambiguity, but there is a possibility that these points, too, may be close to the same circle and add little to the determination. In any case, the value of one of the three rays to A, B and C is lost if P is close to the danger circle.

It is imperative to investigate, during the reconnaissance phase of the survey, whether the resection point is well away from the danger circles formed by each grouping of three of the control points used. Unless this question is explicitly examined, it is easy to overlook a dangerously insecure determination of a point. The uncertainty will not necessarily show up in the computations.

Because of this hidden danger, serious consideration should always be given to adding extra observations in the form of one or more forward bearings or distances to the unknown point P.

Example

An example of a resection from five control points appears on pages *9* and *10* of the *Numerical Example*.

Note The rays drawn in the graph, page *10*, do not have the same significance as in a normal error figure. In the graph of a normal intersection (eg pages *7*, *8*) each ray is a locus and the final point should be as close as possible to all rays. In a resection graph, on the other hand, the equivalent locus is the tangent to the circle which subtends the angle measured between each pair of stations. Four such tangents are drawn in the graph. One way of drawing the tangents is, first, to indicate a set of "shadow rays" (dashed lines in graph) each of which is parallel to the original ray but displaced by an amount which is proportional to the length of the ray. In this case all the displacements are in an anticlockwise (negative) sense and equivalent to a swing of $-0.6''$. On the ray of length 14331 m, the displacement is

$$-\left(\frac{0.6}{206\,000} \times 14\,331\right) = -0.042 \text{ m}$$

Having graphed the shadow rays, the tangents can be drawn in by joining up corresponding intersections of the original and the "shadow rays" (double lines in graph). Each tangent is a locus, the *tangents* form the error figure and the final point must be chosen as close as possible to all tangents. The example is unusual in that the tangents all pass through the same point. Selection of the final position in the error figure can become a problem, particularly if the number of rays is large. For three rays, the number of tangents is three, and they all pass through a single point. For four rays, there are six tangents; for five rays, ten tangents and for six rays, fifteen. It becomes necessary to select the more important intersections and to draw only the tangents corresponding to these intersections.

10.7 *Area from Co-ordinates*

Formulae

If a figure has corner points with known co-ordinates then the area A can be calculated from one of the formulae (10.18) or (10.19) below. The other can then be used as a check.

The figure has n corner points numbered *1* to n in clockwise order. Equation (10.18) should give a negative result.

$$A = \tfrac{1}{2}\{E_1(N_2 - N_n) + E_2(N_3 - N_1) + E_3(N_4 - N_2) + \ldots + E_n(N_1 - N_{n-1})\} \qquad \ldots(10.18)$$

$$A = \tfrac{1}{2}\{N_1(E_2 - E_n) + N_2(E_3 - E_1) + N_3(E_4 - E_2) + \ldots + N_n(E_1 - E_{n-1})\} \qquad \ldots(10.19)$$

Example

The figure *C.D.2.F.8.* in section 12 (figure 11, page *12*). The co-ordinates are copied from the *Co-ordinate List*, page *11*. $n = 5$

Point No.		E	N
5		+ 127 162.05	+ 25 574.16
1	*C*	+ 127 009.76	+ 24 783.65
2	*D*	+ 125 409.16	+ 25 092.10
3	2	+ 125 561.63	+ 25 883.11
4	*F*	+ 125 759.26	+ 25 844.37
5	8	+ 127 162.05	+ 25 574.16
1		+ 127 009.76	+ 24 783.65

$$2A = -2\,568\,764.50 = 2\,568\,764.5$$

$$A = 1\,284\,382.25 \text{ m}^2$$

$$= 128.4382 \text{ Ha}$$

11. NUMERICAL EXAMPLE—CO-ORDINATION OF A CONTROL SURVEY BY SEMI-GRAPHIC METHODS AND CONNECTED PROPERTY SURVEY: EXPLANATION

11.1 *Introduction*

As an example to illustrate the details of co-ordinate computations, a high precision survey is calculated by semi-graphic methods, the calculations being shown in section 12. In the present section a detailed explanation and commentary is given. Section 12 forms a separate unit, with pages numbered from *1* to *18* (Italic script numerals) at the top, and references in sections 11 and 12 are to these page numbers. Each numbered page contains the material which would appear on one handwritten sheet of calculations, though in some cases this takes up more than one page in the *Manual*.

The example is a hypothetical one and several features have been chosen so as to provide the best form of illustration; it is calculated to high precision (0.001 m and 0″.1); it is calculated by semi-graphic methods even though such a survey would normally be calculated by least squares methods on an electronic computer; and very large values have been adopted for the Eastings in order to increase the size of the projection corrections.

In a survey computed by semi-graphic methods, the points to be fixed must be calculated in a specific sequence as each point, as it has been calculated, must be able to be used as a fixed point for the calculation of subsequent points in the sequence.

This requires, therefore, that the reconnaissance is carried out so that each point stands on its own in the sequence of calculation. This point is brought out in the *Plan of Triangulation* (page *2*) where *X* is fixed first, *Y* second, using *X* for this purpose, and finally *Z*, using both *X* and *Y* as fixed points.

In practice, this often means that many more redundant observations must be made than is necessary for the least squares method of calculation, where the fixing of all the points is carried out simultaneously and not sequentially.

11.2 *Preliminary—Abstract of Observations*

To facilitate standardization, angular measurements are made on *n* arcs with *n* zeros, where *n* is 2, 4, 8 or 16 depending on the precision required. The circle is shifted between each arc by $(180° + V)/n$ where V is the micrometer length of the theodolite used e.g. 10′ for a Wild T2. (*See* table VII).

Each arc is meaned and all subsequent arcs have such a constant applied to the mean that each is reduced to the same value as the first observation of the first arc. This practice makes comparison easy. The grand mean is then used in calculation.

It is common practice in Australia to start the first arc with a value near zero (table VII, method 2). There are also advantages in setting the first reading of the first arc to a value close to the bearing of this line. This has been illustrated in the numerical example (table VII, method 1). This practice results in orienting swings which are small quantities (seconds only), easily applied, thus avoiding the awkward operations of adding and subtracting degrees, minutes and seconds.

11.3 *Preliminary—Eccentric Observations*

When it is not possible to set up over a control point, observations are taken instead from a position as near as possible to the point. This nearby position is called an eccentric station or satellite station.

TABLE VII
Example of Field Observations

METHOD 1: Observations in Approximate Bearing

AT	FL	FR	FR−FL	MEAN
*P*1	ARC I			
*P*2	263° 15′ 06.5″	83° 15′ 13.9″	+7.4″	263° 15′ 10.2″
*P*3	292 44 00.9	112 44 10.5	+9.6	292 44 05.7
X	15 13 57.4	195 14 05.4	+8.0	15 14 01.4
	ARC II			−45 −2 −21.3
*P*2	308 17 29.0	128 17 34.0	+5.0	308 17 31.5
*P*3	337 46 22.8	157 46 30.0	+7.2	337 46 26.4
X	60 16 21.7	240 16 26.2	+4.5	60 16 24.0
	ARC III			−90 −5 +02.7
*P*2	353 20 03.2	173 20 11.8	+8.6	353 20 07.5
*P*3	22 48 54.6	202 49 04.0	+9.4	22 48 59.3
X	105 18 54.8	295 19 00.0	+5.2	105 18 57.4
	ARC IV			−135 −7 −23.2
*P*2	38 22 30.3	218 22 36.5	+6.2	38 22 33.4
*P*3	67 51 26.8	247 51 37.2	+10.4	67 51 32.0
X	150 21 20.9	330 21 28.4	+7.5	150 21 24.6

Comparison and Combination of Arcs

	I	II	III	IV	Grand Mean
*P*2	263° 15′ 10.2″	263° 15′ 10.2″	263° 15′ 10.2″	263° 15′ 10.2″	263° 15′ 10.2″
*P*3	292 44 05.7	292 44 05.1	292 44 02.0	292 44 08.8	292 44 05.4
X	15 14 01.4	15 14 02.7	15 14 00.1	15 14 01.4	15 14 01.4

METHOD 2: Observations on 0° Reference

AT	FL	FR	FR−FL	MEAN
*P*1	ARC I			−10.2
*P*2	00° 00′ 06.5″	180° 00′ 13.9″	+7.4	00° 00′ 10.2″
*P*3	29 29 00.9	209 29 10.5	+9.6	29 29 05.7
X	111 58 57.4	291 59 05.4	+8.0	111 59 01.4
	ARC II			−45 02 31.5
*P*2	45 02 29.0	225 02 34.0	+5.0	45 02 31.5
*P*3	74 31 22.8	254 31 30.0	+7.2	74 31 26.4
X	157 01 21.7	337 01 26.2	+4.5	157 01 24.0
	ARC III			−90 05 07.5
*P*2	90 05 03.2	270 05 11.8	+8.6	90 05 07.5
*P*3	119 33 54.6	299 34 04.0	+9.4	119 33 59.3
X	202 03 54.8	22 04 00.0	+5.2	202 03 57.4
	ARC IV			−135 07 33.4
*P*2	135 07 30.3	315 07 36.5	+6.2	135 07 33.4
*P*3	164 36 26.8	344 36 37.2	+10.4	164 36 32.0
X	247 06 20.9	67 06 28.4	+7.5	247 06 24.6

Comparison and Combination of Arcs

	I	II	III	IV	Grand Mean
*P*2	00° 00′ 00.0″	00° 00′ 00.0″	00° 00′ 00.0″	00° 00′ 00.0″	00° 00′ 00.0″
*P*3	29 28 55.5	29 28 54.9	29 28 51.8	29 28 58.6	29 28 55.2
X	111 58 51.2	111 58 52.5	111 58 49.9	111 58 51.2	111 58 51.2

If, at the eccentric station S of a station P, directions are observed to stations A and B, including the direction and length to station P, the correction x to be applied to the observed directions, for the reduction to P, may be computed by the formula:

$$\sin x = \frac{d}{D} \sin a$$

where D is the distance between P and the observed station and a is the angle reckoned clockwise from SP to the observed station, the sign of the correction depending on the sign of $\sin a$.

Assuming that x in radians $= \sin x$, for small angles

$$x'' = \frac{206\,265'' \, d}{D} \sin a$$

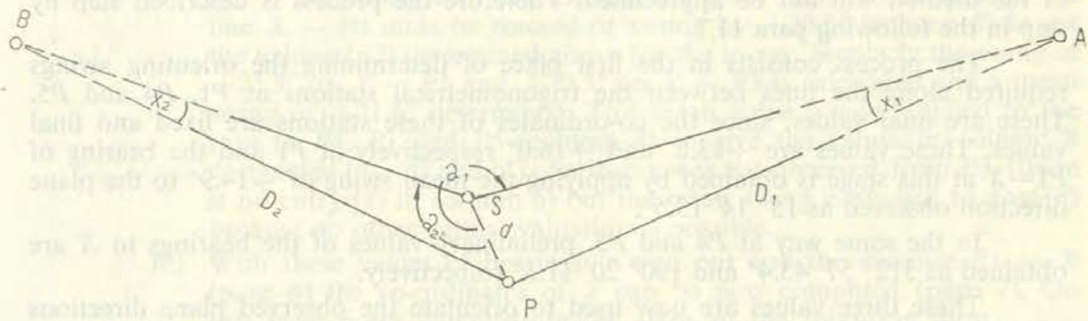


FIGURE 5—REDUCTION OF ECCENTRIC OBSERVATIONS TO CENTRE.

Example

$d = 4.51$ m $206\,265\,d = 930\,300$

Observed		a	$\sin a$	D	x'	x	Reduced
A	0° 0′ 0″.0	284° 49′	−.96675	20 150	−44″.6	0	0° 0′ 0″.0
B	230 15 15.1	155 04	+.42156	15 110	+26″.0	+70″.0	230 16 25.7
P	75 11						

If the reading to station A is zero, it is convenient to reduce x' for this direction to 0, by adding +44.6″ to each x'.

11.4 *The Control Survey—The Co-ordinate List*

The co-ordinate sheet (page *1*) shows the I.S.G. zone of the co-ordinate values used and serves as a ready reference of final co-ordinates as the survey is calculated.

A constant is usually incorporated for convenience. Such constant in any list consists of the figures which are common to all co-ordinate values in the list.

As each point is calculated, its final values are entered on the co-ordinate list and when this value is required in further calculation, it is taken from the co-ordinate list. This practice ensures correct transcription of values.

11.5 *The Control Survey—Projection Corrections*

In order to reduce the observations to the plane of projection it is necessary to have approximate values of the co-ordinates of the stations. Generally, sufficient accuracy is obtained from a plan of the survey to a suitable scale as shown in the attached example (Page *2*). If higher accuracy is necessary, approximate co-ordinates can be calculated from the unreduced observations.

The orientation data, scale factor and arc-to-chord corrections are shown in the example under *Miscellaneous Calculations* (pages *3* and *4*). The application of scale factor and arc-to-chord corrections is carried out on the *Abstract Sheet* as shown in the example (pages *5* and *6*). Later calculations follow in the same manner.

11.6 *The Control Survey—Direction Sheet*

The orientation of observed directions is a process of iteration, as is shown in the example (pages *5* and *6*). In this para 11.6, the process is described briefly. However it is essential to master the steps of the compilation, otherwise the simplicity of the method will not be appreciated. Therefore the process is described step by step in the following para 11.7.

The process consists in the first place of determining the orienting swings required along the lines between the trigonometrical stations at *P*1, *P*4 and *P*5. These are final values, since the co-ordinates of these stations are fixed and final values. These values are +13.8″ and +16.0″ respectively at *P*1 and the bearing of *P*1 − *X* at this stage is obtained by applying the mean swing of +14.9″ to the plane direction observed as 15° 14′ 13.5″.

In the same way at *P*4 and *P*5, preliminary values of the bearings to *X* are obtained as 312° 57′ 43.4″ and 190° 20′ 11.5″ respectively.

These three values are now used to orientate the observed plane directions at *X*. They give rise to preliminary orienting swings of 20.5″, 23.9″ and 20.3″. The mean of 21.6″ is applied to the observed values and this gives seven bearings from which the position of *X* is to be determined. These are—

*P*1.*X*	15° 14′ 13.5″	*X*.*P*1	195° 14′ 11.2″
*P*4.*X*	312 57 43.4	*X*.*P*4	132 57 44.5
*P*5.*X*	190 20 11.5	*X*.*P*5	10 20 12.8
		X.*P*3	271 21 30.6

The mean of the bearings of each line as used for this purpose is shown on the page headed "*To determine X*" (page *7*).

11.7 *The Control Survey—An Explanatory Note on the Direction Sheet*

Section 12, page *5* shows all of the calculations on a direction sheet, but unless the steps of compilation are mastered, the simplicity of the method will not be appreciated.

The principle is to derive the best values for the directions by a process of taking means.

(a) In column 2 the Observed Directions are abstracted from the field book; arc-to-chord corrections are entered in column 3 and applied to the values of column 2 to give the Observed Plane Directions of column 4.

(b) For the line *P*1 − *P*2, the Observed Plane Direction of 263° 15′ 11.2″ (column 4) must be corrected by +13.8″ (column 9) to give the control or datum value of 263° 15′ 25.0″ (column 10).

For the line $P1 - P3$, the corresponding value in column 9 is $+16.0''$. These two values are close enough to suggest no major blunders and a mean of $+14.9''$ is taken as applicable to the third line $P1 - X$, and entered in column 7. This is applied to the observed plane direction of 15° 13′ 58.6″ (column 8) which is designated as (*1*) on the page.

(c) Similarly at $P4$ and $P5$ the mean correction, which is an orienting swing of observed directions, is calculated and applied to give the directions designated as (*2*) and (*3*).

(d) The first station to be located is, in this case, Station X, and the values of (*1*), (*2*) and (*3*) above determined are carried down to column 6 for Station X and entered as *reverse* directions. At this stage we have the best estimate of the three directions *into* Station X as derived from control stations. But at Station X, *four* rays are observed *out*, three of these being to the above three stations, and to adjust these *four* rays the best estimate by the direction method is to orient the values at X to accord with a mean value of the *three* rays observed inward.

The Observed Plane Direction 132° 57′ 22.9″ (column 4) of the line $X - P4$ must be rotated or swung by $+20.5''$ (column 5) to get the value of (*2*) determined above for the in-ray. Similarly the swing of $+23.9''$ on $X - P1$ and $+20.3''$ on $X - P5$ is calculated and a mean swing of $+21.6''$ determined. This mean value is now applied to the rays to $P4$, $P1$ and $P5$ (column 4) to give the values in column 8 designated (*5*), (*6*) and (*7*). An in-ray was *not* observed from $P3$, (there is no entry (*4*) in column 6) but the mean swing is applied to give (*8*) because no other better evaluation is possible.

(e) With these values of bearings in and out and the distance $P1 - X$ (page *6*) the co-ordinates of X can be now computed (page *7*). On this page notes demonstrate the steps in the following form.

		(*1*)	(*5*)
		$\underrightarrow{P1 - X}$	$\underleftarrow{X - P1}$
$\theta P1 - X$	15° 14′ 12.4″	13.5	11.2

The mean direction *in* of $P1$ to $X(1)$ is $\overrightarrow{13.5''}$ (note the arrow)

The mean *reverse* direction of X to $P1$ (5) is $\overleftarrow{11.2''}$ (note the arrow)

The mean direction *in* and *out* is 12.4″

as our best estimate by the process of taking means, and again it appears that there are no gross observational mistakes up to this stage.

(f) With this value for direction and the distance $P1 - X$, (page *6*) preliminary co-ordinates of X are computed. In the three succeeding calculations the cuts of rays from other stations are computed, again using the mean value of the in-rays and the out-rays as oriented earlier. Details of the computation are given in para 11.8.

11.8 *The Control Survey—Computing the Co-ordinates*

The distance $P1.X$ on the projection plane is obtained from the observed value multiplied by the scale factor (pages *3* and *6*).

A preliminary position for X is required and the knotting together of all the observed rays at this point must be shown to a sufficiently large scale. The preliminary position should be one that is sufficiently close to the final position so that no further iteration is required.

For the point *X*, this is most easily determined as a radiation from *P*1. Thereafter, the cuts onto the co-ordinate lines through this preliminary position are calculated as shown in the example. The *larger* co-ordinate difference between a fixed point and the preliminary position of *X* is taken out and it is multiplied by the *lesser* of the tangent or cotangent of the bearing from the fixed point to *X*. The resulting difference is added algebraically to the corresponding value of the fixed point to obtain the point of cut. The distance is also obtained by using the *lesser* of the cosecant or the secant.

The knotting of the rays is then plotted, as illustrated on page *7*, by drawing the rays at the respective directions through these points of cut.

The circle defined by the distance *P*1.*X* is shown as a double line perpendicular to the ray *P*1.*X*.

It is now necessary to find the centre of gravity of the knot point. As long rays are weaker than short ones, each ray is given a weight inversely proportional to its length. The centre of gravity can then be determined with sufficient accuracy by estimation.

Finally, the corrections to the bearings to obtain those to the centre of gravity are obtained by calculating, with slide rule accuracy, the angular swing to produce the scaled displacement from the ray to the final position. These are applied to the calculating bearings to obtain the final bearings.

These are entered in the direction sheet as 15° 14′ 12.6″, 312° 57′ 42.9″, 190° 20′ 12.4″ and 271° 21′ 30.0″ respectively. New orienting means of + 26.6″ at *P*4, −03.6″ at *P*5 and +21.7″ at *X* are taken out for computation of the next point in the sequence of calculation.

This whole cycle is then repeated for the next point; in this example, *Y* (page *8*). Here the preliminary position for *Y* is determined from the intersection of the two rays *P*4−*Y* and *P*5−*Y*.

11.9 *The Control Survey—The Resection*

The resection for the point *Z* (page *9*) is shown to illustrate its calculation by means of one of the many methods of solutions. It is advantageous to calculate the resection to get a preliminary position close to the final one rather than to do a graphical solution, followed often by several iterations, to obtain the final value.

The observations at *Z* were not observed in bearing, but to keep the swing from being very large, the bearing was estimated in the field and the circle set accordingly. Orientation was computed along the longest ray *X*—*P*3 and after the observed plane directions at *Z* were oriented, the preliminary position of *Z* was taken out from an intersection of the other two rays *P*1−*Z* and *X*−*Z*. A check on *calculation* was obtained by the cut of the third ray *P*3−*Z* passing exactly through the preliminary value.

The check on the *fix* of *Z* was obtained from agreement of the redundant rays *Y*−*Z* and *P*4−*Z*. The centre of gravity of a resection fix is obtained in a different manner from that of a point located by intersection in which the orientation is well determined and checked by forward rays in to the point. In the resection, the orientation is unknown and must be determined. If the orientation is altered by a constant, the rays will displace by amounts which are proportional to their lengths. The circle defined by the observed angle subtended at *Z* by two fixed points is, however, not altered by a change in orientation of the directions. Certain of the tangents to the circles are shown in the example of the error figure at *Z* by means of double lines. The tangents will not necessatily knot together as well as occurs in this example. The centre of gravity of the knot of the tangents is determined by estimation.

Finally the corrections to the calculating bearings are taken out and the final bearings determined and entered at *Z* in the *Direction Sheet* (pages *5* and *6*).

11.10 *Selection of the Final Point in the Error Figure*

The error figures for *X* and *Y* are comparatively simple. More complex cases arise which require careful evaluation. The error figure in a resection requires special consideration. A statement made by Dixon will be appreciated sooner or later by any person determining positions by the direction or semigraphical method. "*......when there are more than three rays it is not in general possible to choose a solution which satisfies all the conditions of weights, and the choosing of a final point becomes to a certain extent a matter of opinion. The only general rule which I have so far been able to apply in such differences is that since someone has to make a decision, the opinion of the head of the computing section shall carry the most weight. However it would seem that the soundest principle to work on is that the final point should be as close as possible to the shortest rays and distant from them in proportion to their lengths*".

It is also worth noting as a reassurance, that given good survey data and sound control, there will be little difference in the final selection of a point by any two computers.

This matter is not pursued any further in the *Manual*, though references in the *Bibliography* are provided for further reading.

11.11 *Survey for fixing the property corners—calculation of the subsidiary traverse*

The following paras (11.11-11.13) describe the section of the *Example* which is most typical of normal integrated surveys. It is not discussed in detail as the methods are familiar. The subsidiary traverse (*See* figure 11 on page *12*) was carried out to provide control points from which the corner points of portions 118 and 141 could be fixed conveniently. These computations commence on page *11*.

The angular misclosure of the traverse was first taken out and adjusted (page *13*) then the co-ordinate differences were calculated and adjusted by means of the Bowditch Rule and finally the adjusted co-ordinates taken out (page *15*).

11.12 *Property survey—fixing and checking the corner points*

The boundary corner *F* was fixed by radiation from traverse station *Q*. It was checked by observing a set of directions from *F* to surrounding fixed points. (page *16*).

The boundary corner *E* was fixed, also by radiation, from traverse station *P*. To check it, a mark was set on line from *P* to *Z*. Directions observed at this mark were observed to prove that the mark was on the line *PZ* and, by a side ray to the trig station *P*1, to test the distance from *P* to the mark. The first radiation was then checked by an independent one from the mark.

11.13 *Property survey—co-ordination of portions 118 and 141 on the I.S.G.*

The survey of lots 118 and 141 had been carried out previously. From the data of this survey, three traverses, a northern one, a central one and a southern one, were calculated from *E* to *F*. For each of these, the distance and bearing between *E* and *F* was taken out with the following results:—

Traverse	Bearing E—F	Distance EF
Northern	*272° 54′ 50″*	*7109.347 links*
Central	*272 55 16*	*7108.819*
Southern	*272 54 53*	*7110.287*
Mean	*272° 55′ 00″*	*7109.484 links*

From the I.S.G. co-ordinates of E and F, the corresponding bearing and distance in metres is taken out on the I.S.G.

This gives the following:

I.S.G. Bearing EF 283° 45′ 25″ Distance 1430.3483m.

The difference of 10° 50′ 25″ between the bearings of the common line on the two systems is used to swing the previous survey on to the I.S.G. system and the ratio between distances on the two systems is used to convert the link distance to the I.S.G. metre distances.

$$\text{Mean Ratio} = \frac{1430.3483}{7109.484} = 0.20118\,88.$$

With these converted values the three traverses are now recalculated and adjusted in the usual manner by the Bowditch Rule (pages *17* and *18*). These values are shown with the misclosures as well as the adjusted co-ordinate values of the corner points of the two properties on the I.S.G.

12. NUMERICAL EXAMPLE OF THE CO-ORDINATION OF A CONTROL SURVEY BY SEMIGRAPHIC METHODS AND CONNECTION OF A PROPERTY SURVEY TO THE CONTROL

CONTROL SURVEY

LAND BOARD DISTRICT .... DIVISION

based on I.S.G. Zone 55/3 (Central Meridian 149° E.) Trig. Control

Contents

SUBSIDIARY SURVEY FOR FIXING PROPERTY CORNERS PORTIONS 118 AND 141

COUNTY OF .... PARISH OF..........

Contents

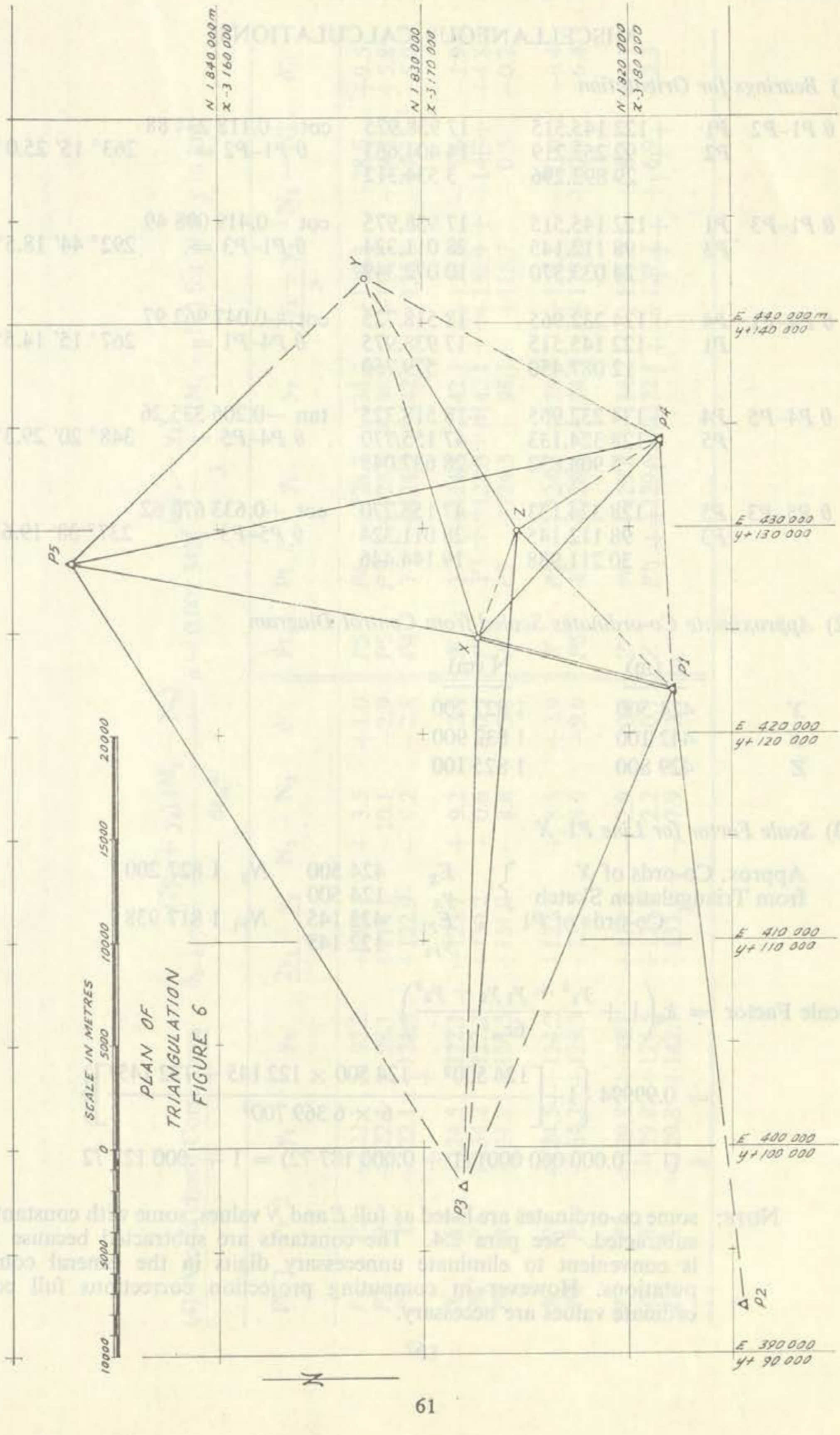
CO-ORDINATE LIST

I.S.G. Zone 55/3

Central Meridian 149° E.

Page	Station	E (m.)	N (m.)	Description of Marks
	Constant	+300 000.000	+1 800 000.000	
	*P*1	+122 145.515	+ 17 938.975	Trig. Station
	*P*2	+ 92 253.219	+ 14 404.663	Trig. Station
	*P*3	+ 98 112.145	+ 28 011.324	Trig. Station
	*P*4	+134 232.965	+ 18 518.725	Trig. Station
	*P*5	+128 324.133	+ 47 155.770	Trig. Station
7	*X*	+124 717.234	+ 27 380.468	G.I.P. in concrete
8	*Y*	+142 150.078	+ 32 870.320	G.I.P. in concrete
9, 10	*Z*	+129 875.965	+ 25 472.963	G.I.P. in concrete

NOTE: The example is a hypothetical one. The Eastings have been increased in order to create the situation where all projection corrections become large enough to be significant. This illustrates more effectively the principles involved.



PLAN OF TRIANGULATION
FIGURE 6

MISCELLANEOUS CALCULATIONS

(1) *Bearings for Orientation*

θ *P*1–*P*2	*P*1	+122 145.515	+17 938.975	cot +0.118 234 88	
	*P*2	+ 92 253.219	+14 404.663	θ *P*1–*P*2 =	263° 15′ 25.0″
		− 29 892.296	− 3 534.312		
θ *P*1–*P*3	*P*1	+122 145.515	+17 938.975	cot −0.419 098 49	
	*P*3	+ 98 112.145	+28 011.324	θ *P*1–*P*3 =	292° 44′ 18.5″
		− 24 033.370	+10 072.349		
θ *P*4–*P*1	*P*4	+134 232.965	+18 518.725	cot +0.047 962 97	
	*P*1	+122 145.515	+17 938.975	θ *P*4–*P*1 =	267° 15′ 14.5″
		− 12 087.450	− 579.750		
θ *P*4–*P*5	*P*4	+134 232.965	+18 518.725	tan −0.206 335 26	
	*P*5	+128 324.133	+47 155.770	θ *P*4–*P*5 =	348° 20′ 29.3″
		− 5 908.832	+28 637.045		
θ *P*5–*P*3	*P*5	+128 324.133	+47 155.770	cot +0.633 670 52	
	*P*3	+ 98 112.145	+28 011.324	θ *P*5–*P*3 =	237° 38′ 19.6″
		− 30 211.988	−19 144.446		

(2) *Approximate Co-ordinates Scaled from Control Diagram*

	E (m)	N (m)
X	424 500	1 827 200
Y	442 100	1 832 900
Z	429 800	1 825 100

(3) *Scale Factor for Line P1–X*

Approx. Co-ords of *X* from Triangulation Sketch	E_X	424 500	N_X	1 827 200
	y_X	+124 500		
Co-ords of *P*1	E_{P1}	422 145	N_{P1}	1 817 938
	y_{P1}	122 145		

$$\text{Scale Factor} = k_0\left(1 + \frac{y_1^2 + y_1 y_2 + y_2^2}{6r_m^2}\right)$$

$$= 0.99994\left\{1+\left[\frac{124\,500^2 + 124\,500 \times 122\,145 + 122\,145^2}{6 \times 6\,369\,700^2}\right]\right\}$$

$$= (1 - 0.000\,060\,000)\,(1 + 0.000\,187\,72) = 1 + .000\,127\,72$$

NOTE: some co-ordinates are listed as full *E* and *N* values, some with constants subtracted. See para 2.4. The constants are subtracted because it is convenient to eliminate unnecessary digits in the general computations. However in computing projection corrections full co-ordinate values are necessary.

(4) Arc-to-chord Corrections $\delta_{1-2} = \frac{(2y_1 + y_2)(N_1 - N_2)}{6r_m^2}\rho = 0.002\,542'' \frac{(2y_1 + y_2)}{3}(N_1 - N_2)$ for N & y in km.

Pt. 1	Pt. 2	y_1	y_2	$\frac{2y_1 + y_2}{3}$	$N_1 - N_2$	δ''	Pt. 1	Pt. 2	y_1	y_2	$\frac{2y_1 + y_2}{3}$	$N_1 - N_2$	δ''
*P*1	*P*2	+122.1	+ 92.2	+112.1	+ 3.5	+1.0	*P*5	*P*4	+128.3	+134.2	+130.3	+28.6	+9.5
*P*1	*P*3	+122.1	+ 98.1	+114.1	−10.1	−2.9	*P*5	*P*3	+128.3	+ 98.1	+118.2	+19.2	+5.8
*P*1	*X*	+122.1	+124.5	+122.9	− 9.2	−2.8	*P*4	*Y*	+134.2	+142.1	+136.8	−14.3	−5.0
X	*P*1	+124.5	+122.1	+123.7	+ 9.2	+2.9	*X*	*Y*	+124.5	+142.1	+130.4	− 5.6	−1.9
*P*4	*P*1	+134.2	+122.1	+130.2	+ 0.6	+0.2	*P*5	*Y*	+128.3	+142.1	+132.9	+14.3	+4.8
*P*4	*X*	+134.2	+124.5	+131.0	− 8.8	−3.0	*X*	*P*3	+124.5	+ 98.1	+115.7	− 0.5	−0.2
X	*P*4	+124.5	+134.2	+127.7	+ 8.8	+2.9	*X*	*P*5	+124.5	+128.3	+125.8	−20.0	−6.4
*P*4	*P*5	+134.2	+128.3	+132.2	−28.6	−9.6	*P*5	*X*	+128.3	+124.5	+127.0	+20.0	+6.4
Z	*P*3	+129.8	+ 98.1	+119.2	− 2.9	−0.9	*Z*	*P*4	+129.8	+134.2	+131.3	+ 6.4	+2.1
Z	*X*	+129.8	+124.5	+128.9	− 2.2	−0.7	*Z*	*P*1	+129.8	+122.1	+127.2	+ 6.9	+2.3
Z	*Y*	+129.8	+142.1	+133.9	− 7.9	−2.7							

ABSTRACT OF OBSERVATIONS AND DIRECTION SHEET

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Observed Direction	Arc-to-Chord	Observed Plane Direction	Preliminary Orienting Swing	Preliminary Plane Bearing	Prelim. Orient. Swing	Preliminary Plane Bearing	Final Orienting Swing	Finalised Plane Bearing
At *P*1									
*P*2	263°15′10.2″	+1.0″	263°15′11.2″					+13.8″	263°15′25.0″ [p *3*] (control)
*P*3	292 44 05.4	−2.9	292 44 02.5					+16.0	292 44 18.5 [p *3*] (control)
X	15 14 01.4	−2.8	15 13 58.6			+14.9″	(1) 15 14′13.5″	+14.0	15 14 12.6 [p *7*]
								~~+14.9~~	
At *P*4									
*P*1	267 14 45.9	+0.2	267 14 46.1					+28.4	267 15 14.5 [p *3*] (control)
X	312 57 19.6	−3.0	312 57 16.6			+26.8	(2) 312 57 43.4	+26.3	312 57 42.9 [p *7*]
*P*5	348 20 13.7	−9.6	348 20 04.1					+25.2	348 20 29.3 [p *3*] (control)
Y	28 52 39.4	−5.0	28 52 34.4			+26.6	28 53 01.0 [p *8*]	+26.2	28 53 00.6 [p *8*]
								~~+26.8~~	
								+26.6	
At *P*5									
Y	135 56 10.4	+4.8	135 56 15.2			−03.6	135 56 11.6 [p *8*]	−04.0	135 56 11.2 [p *8*]
*P*4	168 20 22.4	+9.5	168 20 31.9					−02.6	168 20 29.3 [p *3*] (control)
X	190 20 09.0	+6.4	190 20 15.4			−03.9	(3) 190 20 11.5	−03.0	190 20 12.4 [p *7*]
*P*3	237 38 19.0	+5.8	237 38 24.8					−05.2	237 38 19.6 [p *3*] (control)
								~~−03.9~~	
								−03.6	
At *X*									
*P*4	132 57 20.0	+2.9	132 57 22.9	+20.5″	(2) 132 57′43.4″	+21.6	(6) 132 57 44.5	+20.0	132 57 42.9 [p *7*]
*P*1	195 13 46.7	+2.9	195 13 49.6	+23.9	(1) 195 14 13.5	+21.6	(5) 195 14 11.2	+23.0	195 14 12.6 [p *7*]
*P*3	271 21 09.2	−0.2	271 21 09.0		(4)	+21.6	(8) 271 21 30.6	+21.0	271 21 30.0 [p *7*]
*P*5	10 19 57.6	−6.4	10 19 51.2	+20.3	(3) 10 20 11.5	+21.6	(7) 10 20 12.8	+21.2	10 20 12.4 [p *7*]
Y	72 30 52.0	−1.9	72 30 50.1			+21.7	72 31 11.8	+22.0	72 31 12.1 [p *8*]
				mean = +21.6				~~+21.7~~	

NOTE: The underlined bearings in the final column are the controlling bearings derived from the stations of the trigonometrical survey.

ABSTRACT OF OBSERVATIONS AND DIRECTION SHEET (*continued*)

Distance $P1$—X = 9785.315 at mean elevation of 750 metres (Horizontal Distance)
= 9784.163 m (reduced to Sea Level)
= 9784.163 × 1.000 127 72 = 9785.413 m (reduced to Projection)

(1)	(2) Observed Direction (Not in Brg)	(3) Arc-to-Chord	(4) Observed Plane Direction (Not in Brg)	(5) Preliminary Orienting Swing	(6) Preliminary Plane Bearing	(7) Prelim. Orient. Swing	(8) Preliminary Plane Bearing	(9) Final Orienting Swing	(10) Finalised Plane Bearing	
At *Z*										
P3	251°16′59.3″	−0.9″	251°16′58.4″	+23°17′10.3″	274°34′08.7″			+23°17′10.0″	274°34′08.4″	[p *10*]
X	267 00 24.5	−0.7	267 00 23.8		290 17 34.1			+23 17 09.1	290 17 32.9	,,
Y	35 38 18.2	−2.7	35 38 15.5		58 55 25.8			+23 17 09.1	58 55 24.6	,,
P4	124 38 43.6	+2.1	124 38 45.7		147 55 56.0			+23 17 09.0	147 55 54.7	,,
P1	202 27 03.3	+2.3	202 27 05.6		225 44 15.9			+23 17 09.0	225 44 14.6	,,
								09.3		

To Determine X

	mean (1) & (5)	(1) $P1-X$ →	(5) $X-P1$ ←			
θ $P1-X$	15° 14′ 12.4″	13.5	11.2	sin +0.262 8086	$P1$ +122 145.515	+17 938.975 [p *1*]
$P1-X$ = 9785.413 [p *6*]		[p *5*]		cos +0.964 8480	+ 2 571.690	+ 9 441.436
				Preliminary	X +124 717.205	+27 380.411

To calculate cuts at this point for remaining rays

						mean		
$P4$	+134 232.965	[p *1*]	+18 518.725	θ $P4-X$	312° 57′ 44.0″	(2) &(6)	(2) $P4-X$ ↑ 43.4	(6) $X-P4$ ↓ 44.5 [p *5*]
X	+124 717.205							
	− 9 515.760	cot	− 0.931 2831					
csc	− 1.366 4876		+ 8 861.867					
$P4-X$	13 003.168		+27 380.592					

						mean		
$P5$	+128 324.133	[p *1*]	+47 155.770	θ $P5-X$	190° 20′ 12.1″	(3) & (7)	(3) $P5-X$ ↑ 11.5	(7) $X-P5$ ↓ 12.8 [p *5*]
			+27 380.411					
tan	0.182 3925		−19 775.359					
	− 3 606.876	sec	− 1.016 4974					
	+124 717.257	$P5-Z$	20 101.601					

						mean		
$P3$	+ 98 112.145	[p *1*]	+28 011.324	θ $P3-X$	91° 21′ 30.6″	(4) &(8)	(4) $P3-X$ ↑ —	(8) $X-P3$ ↓ 30.6 [p *5*]
	+124 717.205							
	+ 26 605.060	cot	− 0.023 7147					
csc	− 1.000 2812		− 630.932					
$P3-X$	26 612.540		+27 380.392					

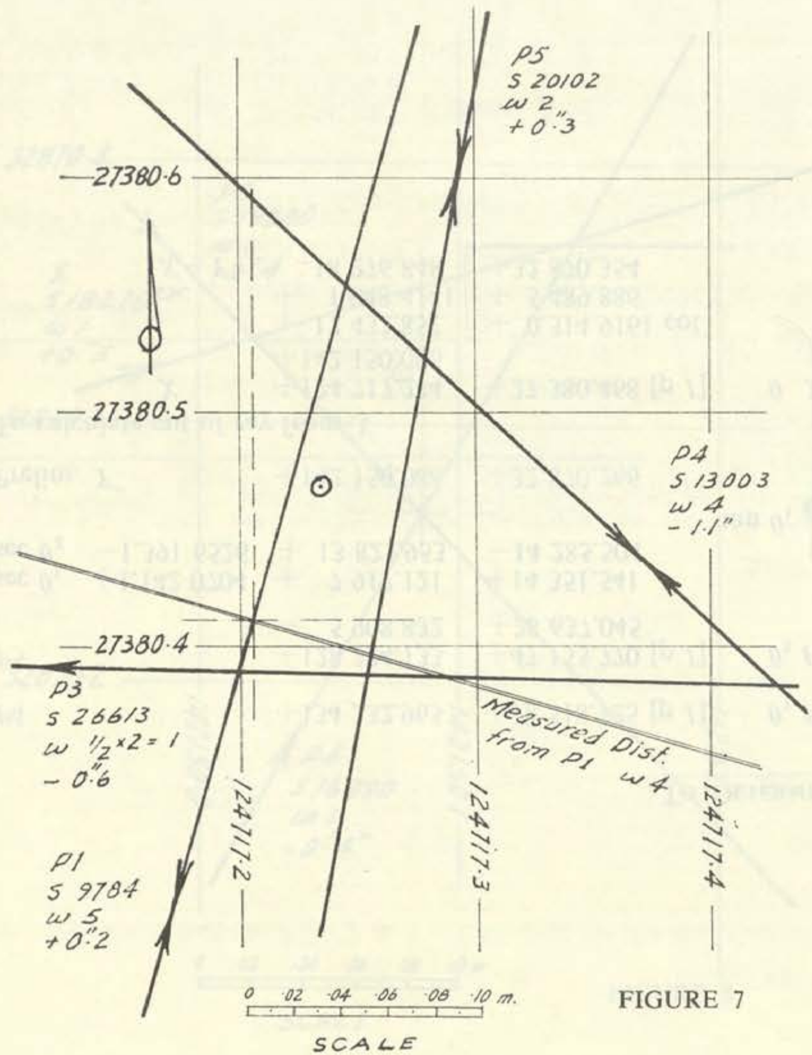


FIGURE 7

Weighted Centre of Gravity at
X+124 717.234 +27 380.468

[Entered p. *1*]

Corrections to Calculating Bearings

Ray	Dis-place-ment	Dist.	Swing	Final Bearing
$P1$—X	.010	9 784	+0.2″	15° 14′ 12.6″
$P3$—X	.076	26 613	−0.6″	91 21 30.0
$P4$—X	.072	13 003	−1.1″	312 57 42.9
$P5$—X	.032	20 102	+0.3″	190 20 12.4

[Entered p. *5*]

NOTE: the swing is computed as

$$\frac{\text{Displacement}}{\text{Distance}} \times 206\,265$$

with the diagram showing the direction

To Determine Y

$P4$		+134 232.965	+18 518.725 [p *1*]	θ_1 $P4-Y$	28° 53′ 01.0″	→ 01.0	← —	[p *6*]
$P5$		+128 324.133	+47 155.770 [p *1*]	θ_2 $P5-Y$	135° 56′ 11.5″	→ 11.6	← —	[p *6*]
		− 5 908.832	+28 637.045					
sec θ_1	+1.142 0704	+ 7 917.121	+14 351.541	tan θ_1	+0.551 6565	s_1	16 390.470	
sec θ_2	−1.391 6526	+ 13 825.953	−14 285.504	tan θ_2	−0.967 8310	s_2	19 880.458	
				tan θ_1 − tan θ_2	+1.519 4875			
Prelim. Y		+142 150.086	+32 870.266					
To calculate cut of ray from X								
X		+124 717.234	+27 380.468 [p *1*]	θ $X-Y$	72° 31′ 11.8″	→ 11.8	← —	[p *5*]
		+142 150.086						
		+ 17 432.852	+ 0.314 9161 cot					
csc		+ 1.048 4141	+ 5 489.886					
$X-Y$		18 276.848	+32 870.354					

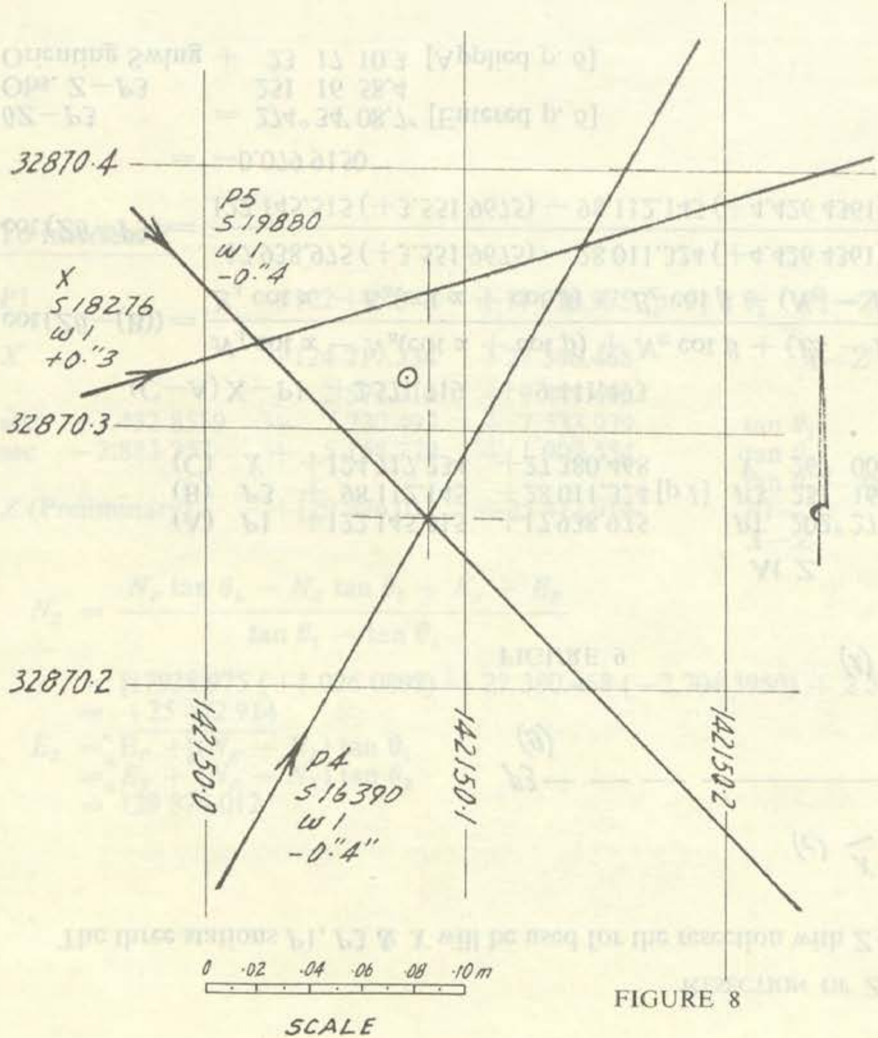


FIGURE 8

Centre of gravity (equal wts)
Y +142 150.078 +32 870.320

[Entered p. *1*]

Corrections to Calculating Bearings

Ray	Dis-place-ment	Dist.	Swing	Final Bearing
$P5-Y$	.037	19 880	−0.4″	135° 56′ 11.2″
$X-Y$	.030	18 276	+0.3″	72 31 12.1
$P4-Y$	.034	16 350	−0.4″	28 53 00.6

[Entered p. *5*]

Resection of Z

The three stations $P1$, $P3$ & X will be used for the resection with $Z-P3$ to give the orientation on the longest line observed

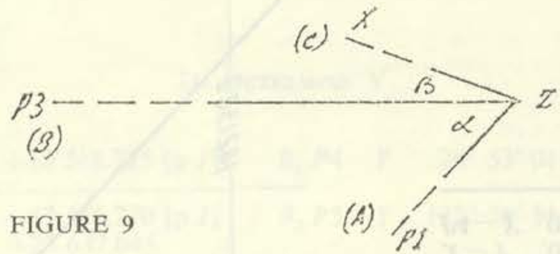


FIGURE 9

				At Z			
(A)	$P1$	+122 145.515	+17 938.975	$P1$	202° 27′ 05.6″ [p. 6]		
(B)	$P3$	+ 98 112.145	+28 011.324 [p 1]	$P3$	251 16 58.4	α 48° 49′ 52.8″	$\cot\alpha$ +3.551 9675
(C)	X	+124 717.234	+27 380.468	X	267 00 23.8	β 15 43 25.4	$\cot\beta$ +0.874 4686
							$\cot\alpha + \cot\beta$ +4.426 4361

(C−A) X−P1 +2 571.719 + 9 441.493

$$\cot(Z\theta-(B)) = \frac{N_A \cot\alpha - N_B(\cot\alpha + \cot\beta) + N_C \cot\beta + (E_C - E_A)}{E_A \cot\alpha - E_B(\cot\alpha + \cot\beta) + E_C \cot\beta - (N_C - N_A)}$$

$$\cot(Z\theta-P3) = \frac{17\,938.975\,(+3.551\,9675) - 28\,011.324\,(+4.426\,4361) + 27\,380.468\,(+0.874\,4686)\,(+2\,571.719)}{122\,145.515\,(+3.551\,9675) - 98\,112.145\,(+4.426\,4361) + 124\,717.234\,(+0.874\,4686)\,(+9\,441.493)}$$

$$= -0.079\,9150$$

$\theta Z-P3$	= 274° 34′ 08.7″	[Entered p. 6]
Obs. $Z-P3$	251 16 58.4	
Orienting Swing +	23 17 10.3	[Applied p. 6]

<u>To intersect Z</u>

$P1$		+122 145.515	+17 938.975 [p. *1*]	θ_1	$P1-Z$	45° 44′ 15.9″	$\rightarrow$ — $\leftarrow$	15.9 [p. *6*]
X		+124 717.234	+27 380.468		$X-Z$	110 17 34.1	$\rightarrow$ — $\leftarrow$	34.1 [p. *6*]
		+ 2 571.719	+ 9 441.493					
sec	+1.432 8519	+ 7 730.497	+ 7 533.939	tan θ_1		+1.026 0898		
sec	−2.883 357	+ 5 158.779	+ 1 907.554	tan θ_2		−2.704 3950		
				tan θ_1 − tan θ_2		+3.730 4848		
Z (Preliminary)		+129 876.012	+25 472.914	$P1-Z$		10 795.02		
				$X-Z$		5 500.16		

$$N_Z = \frac{N_P \tan\theta_1 - N_X \tan\theta_2 + E_X - E_P}{\tan\theta_1 - \tan\theta_2}$$

$$= [17938.975\,(+1.026\,0898) - 27\,380.468\,(-2.704\,3950) + 2\,571.719]/3.730\,4848$$

$$= +25\,472.914$$

$$E_Z = E_P + (N_Z - N_P)\tan\theta_1$$

$$= E_X + (N_Z - N_X)\tan\theta_2$$

$$= 129\,876.012$$

To calculate cuts at Z +129 876.012 +25 472.914

							→	←
*P*3	+ 98 112.145		+28 011.324 [p. *1*]		*0 P*3–*Z*	94° 34′ 08.7″	—	08.7 [p. *6*]
	+129 876.012							
	+ 31 763.867	cot	– 0.079 9150					
csc	1.003 184		– 2 538.409					
*P*3–*Z*	31 765.00		+25 472.915					

N.B.: This check shows that resection up to this point has been correctly calculated as the three rays of the resection itself must pass through one point as they produce a unique solution with no check on its correctness.

							→	←
Y	+142 150.078		+32 870.320 [p. *1*]		*Y0* –*Z*	238° 55′ 25.8″	—	25.8 [p. *6*]
	+129 876.012							
	– 12 274.066	cot	+ 0.602 6714					
csc	– 1.167 570		– 7 397.229					
Y–*Z*	14 330.83		+25 473.091					
*P*4	+134 232.965		+18 518.725 [p. *1*]		*0 P*4–*Z*	327° 55′ 56.0″	—	56.0 [p. *6*]
			+25 472.914					
tan	– 0.626 5155		+ 6 954.189					
	– 4 356.907	sec	+ 1.180 051					
	+129 876.058	*P*4–*Z*	8 206.30					

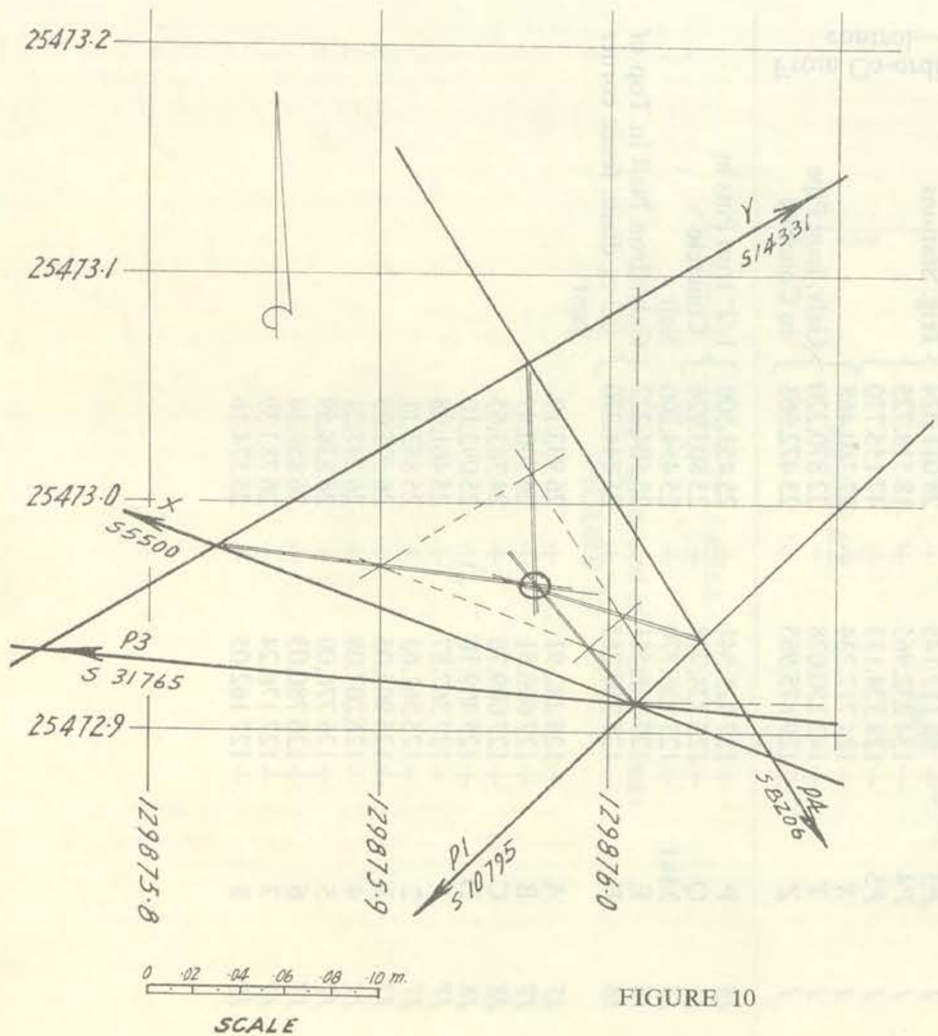


FIGURE 10

Legend for Tangent Method

(valid only for Resection)

——— Calculated Rays
- - - - - - Shadow Rays with displacement proportional to length of ray
═══ Tangents

NOTE: This is not a full tangent solution.

Centre of Gravity of Tangents
Z +129 875.965 +25 472.963

[Entered p. *1*]

Corrections to Calculating Bearings

Ray	Displacement	Dist.	Swing	Final Bearing
X−*Z*	.033	5 500	−1.2″	110° 17′ 32.9″
*P*3−*Z*	.046	31 765	−0.3″	94 34 08.4
*P*1−*Z*	.070	10 795	−1.3″	45 44 14.6
*P*4−*Z*	.052	8 206	−1.3″	327 55 54.7
Y−*Z*	.082	14 331	−1.2″	238 55 24.6

[Entered p. 6]

SURVEY FOR FIXING THE PROPERTY CORNERS

OF

PORTIONS 118 AND 141

COUNTY OF........ PARISH OF........

LAND BOARD DISTRICT........, ...DIVISION

On Integrated Survey System

Zone 55/3

CO-ORDINATE LIST

I.S.G. Zone 55/3

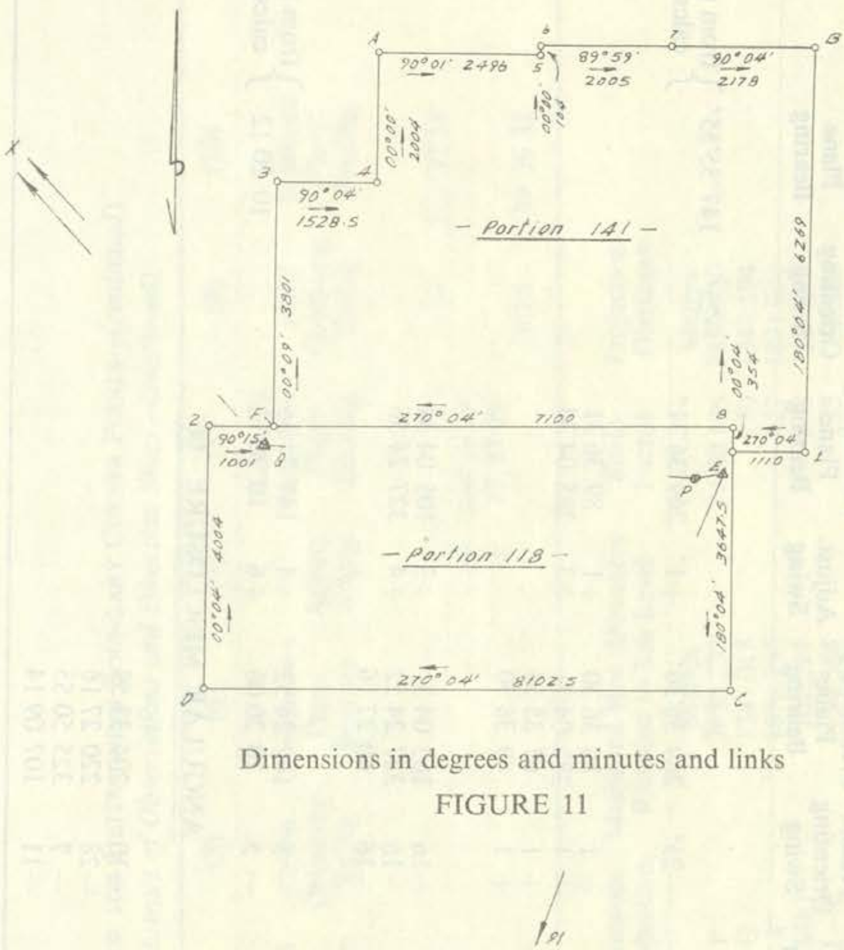
Central Meridian 149° *E*

Page	Station	*E* (m)	*N* (m)	Description of Marks	
	Constant	+300 000.000	+1 800 000.000		
1	*P*1	+122 145.515	+ 17 938.975	Trig. Stations	From Co-ordinate List of control—page *1*
1	*P*2	+ 92 253.219	+ 14 404.663	Trig. Stations	From Co-ordinate List of control—page *1*
1	*P*3	+ 98 112.145	+ 28 011.324	Trig. Stations	From Co-ordinate List of control—page *1*
1	*P*4	+134 232.965	+ 18 518.725	Trig. Stations	From Co-ordinate List of control—page *1*
1	*P*5	+128 324.133	+ 47 155.770	Trig. Stations	From Co-ordinate List of control—page *1*
1	*X*	+124 717.234	+ 27 380.468	Galv. Iron Pipe in Concrete	From Co-ordinate List of control—page *1*
1	*Y*	+142 150.078	+ 32 870.320	Galv. Iron Pipe in Concrete	From Co-ordinate List of control—page *1*
1	*Z*	+129 875.965	+ 25 472.963	Galv. Iron Pipe in Concrete	From Co-ordinate List of control—page *1*
15	P	+127 025.541	+ 25 453.509	1/2″ Iron Pins in Concrete	
15	Q	+125 724.778	+ 25 803.926	1/2″ Iron Pins in Concrete	
15	Mk1	+127 126.770	+ 25 454.200	Nail	
16	E	+127 148.581	+ 25 504.227	Galv. Iron Nail in Top of 0.3 m diam. fence corner post	
16	F	+125 759.265	+ 25 844.370	Galv. Iron Nail in Top of 0.3 m diam. fence corner post	
17	A	+126 282.91	+ 26 933.19		
17	B	+127 606.51	+ 26 700.47		
18	C	+127 009.76	+ 24 783.65		
18	D	+125 409.16	+ 25 092.10		
17	1	+127 367.87	+ 25 461.98		
17	2	+125 561.63	+ 25 883.11		
17	3	+125 905.05	+ 26 595.05		
17	4	+126 207.08	+ 26 537.21		
17	5	+126 776.09	+ 26 838.59		
17	6	+126 780.03	+ 26 859.14		
17	7	+127 176.24	+ 26 783.39		
18	8	+127 162.05	+ 25 574.16		

PLAN OF PORTIONS 118 AND 141

COUNTY OF........ PARISH OF........

LAND DISTRICT........ LAND BOARD DISTRICT........ ......DIVISION



Dimensions in degrees and minutes and links

FIGURE 11

TRAVERSE FOR DETERMINING BOUNDARY CORNER POINTS

Abstract of Observations and Direction Sheet

(1)	(2) Observed Direction	(3) Arc-to-Chord	(4) Observed Plane Direction	(5) Prelim. Orienting Swing	(6) Preliminary Plane Bearing	(7) Adjust. Swing	(8) Adjusted Preliminary Plane Bearing	(9) Final Orienting Swing	(10) Finalised Plane Bearing	
AT *Z*										
*P*4	147°56′18″	+2″	147°56′20″					−25″	147°55′55″	from preceding calcs. p. *6*
P	269 36 55	0	269 36 55	−25″	269°36′30″	+1″	269°36′31″			
AT P										
Z	89 36 29	0	89 36 29	+ 1	89 36 30	+1	89 36 31			
Q	285 04 30	0	285 04 30	+ 1	285 04 31	+3	285 04 34			
E	67 35 54	0	67 35 54	+ 1	67 35 54					
Mk1	89 36 29	0	89 36 29	+ 1	89 36 30					
AT Q										
P	105 04 47	0	105 04 47	−16	105 04 31	+3	105 04 34			
X	327 25 09	−1	327 25 08	−16	327 24 52	+4	327 24 56			
F	40 27 32	0	40 27 32	−16	40 27 16					
AT *X*										
Q	147 24 56	+1	147 24 57	− 5	147 24 52	+4	147 24 56			from preceding calcs p. *5*
*P*5	10 20 17	−6	10 20 11	− 5	10 20 06	+6	10 20 12		10 20 12	
					ANGULAR MISCLOSURE−06″					
AT F										
*P*1	204 34 07	+2″	204 34 09	−10	204 33 59					
Q	220 27 44	0	220 27 44	−28	220 27 16					
X	325 51 02	0	325 51 02	− 7	325 50 55					
P	107 09 25	0	107 09 25	−11	107 09 14					

TRAVERSE FOR DETERMINING BOUNDARY CORNER POINTS—(*continued*)

Abstract of Observations and Direction Sheet—(continued)

(1)	(2) Observed Direction	(3) Arc-to-Chord	(4) Observed Plane Direction	(5) Prelim. Orienting Swing	(6) Preliminary Plane Bearing	(7) Adjust. Swing	(8) Adjusted Preliminary Plane Bearing	(9) Final Orienting Swing	(10) Finalised Plane Bearing
AT MK1									
*P*1	213°32′30″	+2″	213°32′32″					−18″	213°32′14″
P	269 36 55	0	269 36 55			−16″	269°36′39″		
E	23 33 25	0	23 33 25			−16	23 33 09		
Z	89 36 47	0	89 36 47					−15	89 36 32

Distance Reduction	Measured Hor. Distances Reduced to Sea Level	Scale Factor	Projection Distances
	Metres		Metres
Z—P	2850.232	1.000 136	2850.620
P—Q	1347.018	130	1347.194
Q—*X*	1870.795	127	1871.033
P—Mk1	101.217	133	101.230
Mk1—E	54.564	133	54.570
P—E	133.076	133	133.094
Q—F	53.145	1.000 130	53.151

Calculation and Adjustment of Traverse

Line	Plane Bearing / Proj. Distance	sin / cos	ΔE / Corr. ΔE	ΔN / Corr. ΔN		Adjusted Co-ordinates	
					Z	+129 875.965	+25 472.963
Z—P	269° 36′ 31″	−0.999 9767	−2850.554	− 19.473		− 2 850.424	− 19.454
	2850.620m	−0.006 8310	+ 0.130	+ 0.019			[Entered p. *11*]
					P	+127 025.541	+25 453.509
PQ	285° 04′ 34″	−0.965 5812	−1300.825	+ 350.408		− 1 300.763	+ 350.417
	1347.194	+0.260 1019	+ 0.062	+ 0.009			[Entered p. *11*]
					Q	+125 724.778	+25 803.926
Q*X*	327° 24′ 56″	−0.538 5420	−1007.630	+1576.530		− 1 007.544	+ 1 576.542
	1871.033	+0.842 5986	+ 0.085	+ 0.012			
					X	+124 717.234	+27 380.468
	Σs 6069	$\Sigma\Delta$	−5159.009	+1907.465	Δ	− 5 158.731	+1 907.505
		Misclosure e_E	− 0.278	− 0.040 e_N			

$$\frac{0.281}{6069} = \frac{1}{21600}$$

To fix boundary corner E

Polars					
P—E	67° 35′ 54″	s +0.924 535	P +127 025.541	+25 453.509	
	133.094	c +0.381 097	+ 123.050	+ 50.722	
			E +127 148.591	+25 504.231	Preliminary
			P +127 025.541	+25 453.509	
P—Mk1	89° 36′ 30″	s +0.999 9766	+ 101.228	+ 0.692	
	101.230	c +0.006 8358			
			Mk1 +127 126.769	+25 454.201	[Entered p. *11*]
			+ 2 749.326	+ 18.794	
Mk1—*Z*	89° 36′ 30″				
	2749.390 (by difference)		*Z* +129 876.095	+25 472.995	[OK for check]
			.130	.032	

Joins for orientation and comparison for checking Mk1.

Mk1	+127 126.770	+25 454.200				
*P*1	+122 145.515	+17 938.975	tan +0.662 8218	θ Mk1 − *P*1	213° 32′ 14.2″	
	− 4981.255	− 7515.225			9016.80	
Z	+129 875.965	+25 472.963	cot +0.006 8249	θ Mk1 − *Z*	89° 36′ 32.3″	
	+ 2749.195	+ 18.763				

At Mk 1	Obs. Plane Bearing	Calculated Plane Bearing	Diff.	
*P*1	213° 32′ 14″ (long ray)	213° 32′ 14″	00″	
P	269 36 37	269 36 32	+05″	
E	23 33 07	23 33 07	00″	
Z	89 36 29	89 36 32	−03″	[Check on Mk 1 OK]

To Check E

Mk1—E	23° 33′ 09″	s +0.399 589		Mk1	+127 126.770	+25 454.200	
	54.570	c +0.916 694			+ 21.806	+ 50.024	
			check	E	+127 148.576	+25 504.224	
				E	+127 148.591	+25 504.231	[previous page]
			accept	E	+127 148.581	+25 504.227	[Entered p. *11*]

To fix boundary corner F

			Q	+125 724.778	+25 803.926	
Q—F	40° 27′ 16″	s +0.648 843		+ 34.487	+ 40.444	
	53.151	c +0.760 922				
			F	+125 759.265	+25 844.370	[Preliminary]

Joins for checking

F	+125 759.265	+25 844.370				
X	+124 717.234	+27 380.468	tan	−0.678 3623	θ F*X*	325° 50′ 54.2″
	− 1 042.031	+ 1 536.098				
*P*1	+122 145.515	+17 938.975				
	− 3 613.750	− 7 905.395	tan	+0.457 1245	θ F*P*1	204° 33′ 58.7″
P	+127 025.541	+25 453.509				
	+ 1 266.276	− 390.861	cot	−0.308 6697	θ FP	107° 09′ 13.9″

Check on position of F (preliminary)

Length	At F	Observed Plane Directions	Orienting Swing On Longest Ray	Observed Plane Bearings	Calculated Plane Bearings	Diff.	Displacement
8700	*P*1	204° 34′ 09″	−10″	204° 33′ 59″	204° 33′ 59″	00″	+0.000
53	Q	220 27 44		220 27 34	40 27 16	−18″	−0.005
1900	*X*	325 51 02		325 50 52	325 50 55	+ 3″	+0.030
1320	P	107 09 25		107 09 15	107 09 14	−1″	−0.001

accept F +33 759.265 +11 844.370 [Entered p. *11*]

Traverses Round the Property Boundaries to Obtain I.S.G. Co-ordinates of the Corner Points

Line	Bearing	Distance	sin cos	Preliminary Co-ordinates			Adjusted Co-ordinates	
				+127 148.581	+25 504.227	E	+127 148.581	+25 504.227
E—1	100° 54′ 25″	223.320	+0.981 935 −0.189 215				−0	+4
				+127 367.867	+25 461.972	1	+127 367.867	+25 461.976
1—B	10 54 25	1261.253	+0.189 215 −0.981 935				−3	+27
				+127 606.514	+26 700.441	B	+127 606.511	+26 700.468
B—7	280 54 25	438.189	−0.981 935 +0.189 215				−4	+35
				+127 176.242	+26 783.353	7	+127 176.238	+26 783.388
7—6	280 49 25	403.384	−0.982 210 +0.187 786				−5	+42
				+126 780.034	+26 859.103	6	+126 780.029	+26 859.145
6—5	190 50 25	20.923	−0.188 072 −0.982 155				−5	+42
				+126 776.098	+26 838.553	5	+126 776.093	+26 838.595
5—A	280 51 25	502.168	−0.982 100 +0.188 358				−6	+51
				+126 282.918	+26 933.140	A	+126 282.912	+26 933.191
A—4	190 50 25	403.183	−0.188 072 −0.982 155				−7	+58
				+126 207.091	+26 537.152	4	+126 207.084	+26 537.210
4—3	280 50 25	307.517	−0.982 155 +0.188 072				−7	+63
				+125 905.062	+26 594.987	3	+125 905.055	+26 595.050
3—F	190 59 25	764.719	−0.190 643 −0.981 660				−9	+77
				+125 759.274	+25 844.293	F	+125 759.265	+25 844.370
		Σs 4324	Misclosure	+0.009 (1/55000)	−0.077			

Northern Traverse

[Entered p. *11*]

TRAVERSES ROUND THE PROPERTY BOUNDARIES TO OBTAIN I.S.G. CO-ORDINATES OF THE CORNER POINTS (*continued*)

	Line	Bearing	Distance	sin cos	Preliminary Co-ordinates			Adjusted Co-ordinates		
Central Traverse					+127 148.581	+25 504.227	E	+127 148.581	+25 504.227	[Entered p. *11*]
	E—8	10° 54′ 25″	71.221	+0.189 215 +0.981 935				−7	−4	
					+127 162.057	+25 574.162	8	+127 162.050	+25 574.158	
	8—F	280 54 25	1428.441	−0.981 935 +0.189 215				−157	−74	
					+125 759.422	+25 844.444	F	+125 759.265	+25 844.370	
			Σ1500	Misclosure	+0.157	+0.074				
					(1/8600)					
Southern Traverse					+127 148.581	+25 504.227	E	+127 148.581	+25 504.227	[Entered p. *11*]
	E—C	190 54 25	733.836	−0.189 215 −0.981 935				+37	+2	
					+127 009.728	+24 783.648	C	+127 009.765	+24 783.650	
	C—D	280 54 25	1630.132	−0.981 935 +0.189 215				+118	+6	
					+125 409.045	+25 092.092	D	+125 409.163	+25 092.098	
	D—2	10 54 25	805.560	+0.189 215 +0.981 935				+158	+8	
					+125 561.469	+25 883.100	2	+125 561.627	+25 883.108	
	2—F	101 05 25	201.390	+0.981 325 −0.192 356				+168	+8	
					+125 759.097	+25 844.362	F	+125 759.265	+25 844.370	
			Σ3371	Misclosure	−0.168	−0.008				
					(1/20000)					

PART 3. SURVEYS ON THE INTEGRATED SURVEY GRID

13. BASING SURVEYS ON CONTROL

13.1 *Tying in by Traverse*

The additional requirement, introduced by survey integration, of basing surveys on control points, involves little extra work. This is ensured by the regulations under the Survey Integration Act, which state that if a survey is beyond a specified distance from the nearest control, the connection is not necessary. The benefits and economies arising from working in an integrated system are such that in many cases the surveyor will make the connection even though it is not a legal requirement.

The majority of surveys will be tied in to the control by traverse, as this is the most practical and flexible method of survey for the purpose.

The basic surveying operations of the property, engineering or other survey are unchanged by integration. However some of the changes brought about in surveys in an integration area are listed below.

1. It is necessary to base surveys on the control system.
2. Azimuth is provided automatically by the control system, thus avoiding the need to investigate before adopting a starting azimuth.
3. The survey is related to other work. This will provide, at the very least, some confirmatory evidence. In many cases it enables savings in the work. As more surveys are completed in the integrated system such benefits will increase.
4. Emphasis is on linear traverses rather than loop traverses. The typical form of the survey will be a traverse from a control point, connecting to the survey, and closing onto another control point beyond the survey. In relation to the size of the survey, the extra distances traversed will generally be insignificant, because of the frequent spacing of the control points. In many instances the control will provide a saving by enabling the traverse to close onto a control point near the remote end of the survey, instead of requiring a return traverse to close the loop.
5. Greater emphasis is placed on use of co-ordinates in calculation.

The linear connecting traverse between control points should ideally be oriented by observed directions, one such direction at each end, so that four control points are involved. This provides an independent check that the marks have been properly identified and that they have not been disturbed. If the independent orienting direction at one end is not available, and the control point at the other end is sighted instead, only three points are involved. If no outside points are sighted for orientation, but only the line between the ends of the traverse, the situation is far from ideal. Similarly if the survey starts at a control point and loops back to close onto the same point, from which orienting rays to two or more controls are observed, the situation is unsatisfactory because an error, for example in the co-ordinates of the point, could pass undetected. In the two last cases every effort should be made to obtain an independent check, by sighting onto other control points from intermediate traverse stations, connecting to points on other integrated surveys, or by some other means.

Because of the basic linear shape of many of the surveys there will be a greater tendency to fix points by radiation. As in any other survey, it is essential in the integrated system to apply satisfactory checks to a radiated point. It should be checked by an independent radiation from another point, by a separate "dog-leg" minor traverse, or by other angular and distance measurements so as to provide an independent check on both the field measurements and on the calculation of the co-ordinates of the point.

13.2 *Tying in by edm and combined measurements*

Although the traverse is the most common form of connection from control points to survey, there are circumstances where other forms are appropriate. These occur particularly in rural areas where the distances involved in the surveys are greater. The other forms of survey might include pure triangulation, but are more likely to be some combination of distance and angle measurement.

The traverse, with distances measured by short or medium range edm equipment, can often be complemented by angle measurements to triangulated control points. Similarly, triangulation is often assisted by the generous addition of edm distances.

Probably the most versatile of these forms is the combination of radiation with the distance measured by edm, and resection. The shortcoming of the resection (the danger circle, para 10.6) and the radiation (necessity for checking, para 13.1) are generally avoided in the combination. By observing one or more distances from a resection point, the determination is greatly strengthened, and the versatility of the resection for connections to control, is retained. Viewed differently, a radiation, or a traverse from a control point to the survey, can often be conveniently checked by means of observed directions to a number of control points, thus avoiding the necessity of closing onto control by traverse.

13.3 *Comments on the Example*

The *Numerical Example* of section 12 is designed to illustrate various forms of computation. It is for this reason that there is an extensive survey shown, to bring in control for a comparatively minor property survey. This would not occur in practice, as the operations for connecting the control to the survey would form a far smaller proportion of the total survey. It is likely too, that in practice greater use would be made of edm measurements. The example emphasizes the triangulation methods of intersection and resection because they are less familiar.

One aspect which is however well illustrated by the example is the division into different orders of survey. The survey is based on existing trigonometrical stations, which form the higher order. Triangulation is carried out to break down from the trigonometrical control to the site of the survey (points *X*, *Y* and *Z*). This breakdown survey forms an intermediate order survey, while the property survey, carried out by traverse, is the lower order survey.

13.4 *General Comments*

Surveys in an integrated system involve connections from control to the survey. Unless these connections and the survey are very simple, it is beneficial to undertake a reconnaissance before commencing measurements. During this phase of the survey the control points are located, the surveying technique is chosen in detail, station positions are selected, and intervisibility is checked.

In the design of the survey, linear traverses are preferable to loop traverses. The line traverse is stronger, in the sense that intermediate points on the traverse are fixed more precisely, and provides better checks, particularly in detecting errors of standardization or other proportional errors in the distance measurements.

As the number of surveys on the I.S.G. increases, and the experience of surveyors in an integrated system grows, fuller advantage will be taken of the extra possibilities of checking against other surveys on the common system, and the judicious use of data provided by these surveys.

14. THE CONTROL SYSTEM

14.1 *Orders of Control*

The orders or classes of accuracy for surveys are defined as Classes *A* to *H* in Section 15, where equation (15.2) defines the maximum allowable standard deviation *K* and table VIII gives the value of the parameter *F* for each class of accuracy. *K* depends on the distance *S* between the points being investigated and generally refers to the distance from the point whose accuracy is being tested, to the nearest control point or higher order point. The difference between one accuracy class and the next involves a factor of either 3 or 3⅓. The factor has been made sufficiently large so that it is safe to assume that the higher order points are free of error. The precision of points is expressed in relation to the higher order survey points in the local area.

The classes of accuracy and hence the values of *F* to be adopted for various surveys are specified. Since the specifications may be altered, the description below is in broad terms only and should not be taken as definitive. The ratio quoted is the maximum standard deviation over the distance for fairly long distances, where the effect of the 0.04 term under the root is negligible. It should be borne in mind that occasional errors of twice the standard deviation are acceptable.

CLASS A	Surveys demanding an extraordinarily high precision, ±1½ ppm (parts per million).
CLASS B	First order geodetic surveys, ±5 ppm
CLASS C	Lower order control surveys. Control for survey integration, high precision engineering surveys, ±1/67 000.
CLASS D	Property surveys, engineering surveys, ±1/20 000.
CLASS E	Property surveys in exceptionally difficult terrain, or covering very large areas; lower order engineering surveys, ±1/6 700.
CLASSES F, G, H.	Lower order surveys. G & H are applicable to some topographic and stadia surveys.

In quoting the results of any survey the class of accuracy should also be quoted as this is valuable information for the user of the data. Methods for determining whether the class of accuracy required for a survey is actually achieved, are given in part 4, sections 15 and 17.

14.2 *Availability of Survey Information*

The functions of collection, storage and dissemination of survey data are essential to an integrated survey system. In New South Wales the Department of Lands has the responsibility for these functions. The position in July, 1975, regarding availability of information is summarised below. It is possible that there will be changes as the system evolves to meet the increasing demands for integrated survey information.

Information regarding the location and value of permanent marks, state survey marks, and miscellaneous marks adopted under the Survey Co-ordination Act, and copies of plans lodged in the Central Plan Register, is available on request from the Officer-in-Charge, Survey Co-ordination Branch, Department of Lands, Bridge Street, Sydney. Generally the information is presented in the following forms which are designed for ease of copying.

(a) Control Survey Plans. These are based on the 1:250 000 or 1:4000 map areas, depending on the density of marking, and show the location of each permanent mark, state survey mark and miscellaneous mark on the plan. An accompanying schedule is included which shows, where available, elevations on Australian Height Datum and co-ordinates on the Integrated Survey Grid.

(b) Visual Index Plans. These are on a similar format to the Control Survey Plans, except that the schedule shows details of surveys for which a Notice of Completion and/or Notice of Intention has been lodged (Does not include title surveys).

(c) Recorded Plans. These comprise plans of surveys recorded in the Central Plan Register (Does not include title surveys).

(d) Sketch Plans. These are diagrams of individual marks, showing the relationship of the mark to nearby physical features as an aid to identification in the field.

At present, marks placed do not necessarily have approved A.H.D. and I.S.G. values, and some information is still recorded on an earlier system based on parish maps.

Information on title surveys is available from the Lands Department for Crown lands surveys and from the Lands Title office for other property surveys.

The Department of Lands includes in its records, information on heights and in particular on the Australian Height Datum. Details are given in part 6. In the survey of a boundary defined by mean high water, it may be necessary to consult other authorities. *See* paras 22.3 and 22.6 for details.

PART 4 STANDARDS OF ACCURACY

15. APPLICATION OF ACCURACY STANDARDS IN TRAVERSING

15.1 *Summary*

Before the introduction of regulations under the Integrated Surveys Act it was customary in New South Wales to apply three tests to the measurements of a traverse. These were—

1. Measurement consistency of the individual readings, for example comparison of two measures of an angle.
2. Angular misclose, which was required to be less than

$$30'' + 20''\sqrt{n}$$

where n was the number of angles measured. The maximum allowable misclose was 3 minutes.

3. Linear misclose, which was not to exceed a specified value depending on the type of survey. For example—

$$\text{Misclose} < S/8\,000$$

where S was the length of the traverse.

The dual functions of tests of this type are to ensure that the required precision has been achieved and to guard against gross errors. However the use of individual traverse miscloses is an unreliable and insensitive procedure for achieving these aims and as a result these tests have been replaced with a set designed to investigate the standard deviation of the calculated co-ordinates of the traverse stations. Since the weakest point of a traverse is the mid-point, it is proposed, in order to simplify the testing, to investigate only the mid-point.

The new tests fall into four categories—

1. *Design*. Before any measurements are taken, the proposed field technique is investigated to see whether measurements taken will satisfy the appropriate standard. The standard is defined as

$$K = F\sqrt{0.04 + S^2}\ \text{mm}$$

where F is a factor dependent on the class of traverse and S is the distance from the control station to the mid-point, in kilometres.

The field technique is evaluated to give the standard deviation σ_H of the co-ordinates of the mid-point by the formula:

$$\sigma_H = \sqrt{qS^2\sigma_\theta{}^2 + \tfrac{1}{4}\, r\sigma_S{}^2}$$

where q is a factor from a table which depends on the number of sides and the shape of the traverse,

S is the distance from the end point in kilometres,

σ_θ is the standard deviation of each angle,

σ_S is the standard deviation of each section of taping, and

r is the number of such sections.

For the field technique to be acceptable the requirement is:

$$\sigma_H < K$$

It is not always necessary to examine each individual traverse; a similar traverse may have been investigated previously.

2. *Field consistency*. Measurements in the field should be examined to ensure that they comply with the design values of σ_θ and σ_S.

3. *Angular misclose*. The angular misclose from the measurements must not exceed $2\frac{1}{2}$ times the design value $\sigma_{M\theta}$ where

$$\sigma_{M\theta} = \sigma_\theta . \sqrt{n}$$

4. *Linear misclose*. The linear misclose from the measurements must not exceed $2\frac{1}{2}$ times the design value σ_M (the standard deviation of linear misclose) as calculated from

$$\sigma_M = \sqrt{pS^2\sigma_\theta{}^2 + r\,\sigma_S{}^2}$$

where p is a factor from a table and the other symbols are as before.

Should any test fail, the traverse should be remeasured. To strengthen the surveyor's knowledge of his measuring precision a running check of all tests should be kept for traverses carried out under similar conditions. This will indicate whether reasonable values have been adopted for σ_θ and σ_S for the given field technique.

Although the new procedure cannot claim the same simplicity as the earlier method, it is basically straightforward. Its advantages are that instead of testing quantities only loosely related to precision, it tests the precision of the actual results.

15.2 *Introduction*

The introduction of survey integration brought a need for changes in the standards of accuracy against which to test the results of field surveys. An integrated set of standards is required for surveys made to the various classes of accuracy, including standards for property surveys. The new standards are described below. They are expressed in terms of the standard deviation of a point, rather than a misclose, because the misclose is not a satisfactory indicator of the accuracy of a survey.

The precision of determination of a point is usually expressed in two dimensions in the form of the error ellipse. For convenience, *the standard deviation of a point* is used as a single indicator of precision, derived from the error ellipse. It is defined as the square root of the sum of the squares of the semi-axes of the ellipse.

Since the standard is expressed in terms of the standard deviation, it is necessary to compute the standard deviation of points fixed by traverse. Rigorous least square methods are available but the computations are considerably shorter if simplified formulae are used. Besides the standard deviations of angle and length measurements, the shape of the traverse has a considerable effect on the accuracy of the traverse. In a least squares adjustment it is easy to arrange for the calculation of measures of precision. In the Berthon Jones method, (see *Bibliography*) co-ordinates of the traverse stations are used, and in the simplified method the traverse is classified according to its shape. The two last methods, which are simple to apply, are described in this section.

Significant differences from the previous practice in this new approach are firstly that each surveyor determines the standard deviations of his angle and distance measurements under various circumstances, in much the same was as he standardizes his tape, and secondly that instead of looking at the results of a single survey in isolation, the surveyor uses the results of a whole group of surveys carried out with the same types of equipment in similar circumstances in order to assess the precision of his work. He keeps a running record of these results.

15.3 *Normal Distribution of errors*

If any quantity, a length for example, is measured a number of times, then the various measurements *s* will differ slightly. If the mean of all the measurements is $\overline{S}$ then the deviation of each observation from the mean is *v*, where $v = s - \overline{S}$. If the *v*'s are grouped according to size, we can count the number in each group, for example, 3 between −2.0 and −1.5 cm, 8 between −1.5 and −1.0 cm. In general there are more small errors than large errors, and the distribution of errors can be shown on a graph such as figure 12 (Curve a). If the number of observations is very large and the errors are random, then the curve will tend to take up the smooth bell shape of Curve b in figure 12. This is the *curve of normal distribution* and it is the model against which the actual error distributions present in a set of observations can be tested.

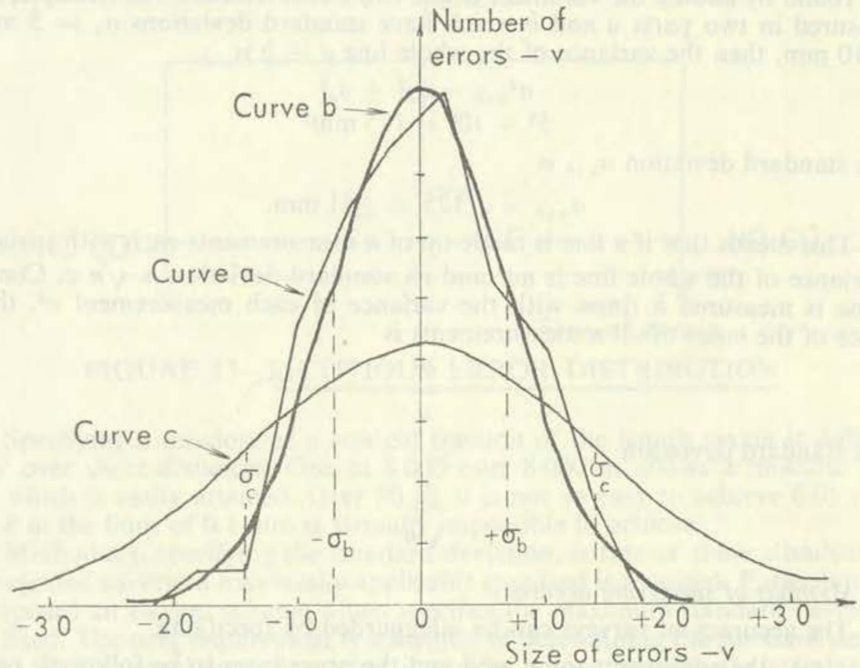


FIGURE 12—DISTRIBUTION OF ERRORS AND NORMAL DISTRIBUTION
Curve (a) Actual error distribution
(b) Theoretical or normal distribution, with standard deviation σ_b
(c) Theoretical or normal distribution, with larger standard deviation σ_c

If two sets of measurements are made, one with a steel tape and the other with a cloth tape, the spread of the distribution curve will be much narrower in the case of the more precise steel tape observations (Curve a) than for the cloth tape observations (Curve c), though in both cases the curve should approximate the characteristic bell shape.

The standard deviation σ is a measure of the spread of the errors—a measure of the width of the bell. It is calculated from the formula

$$\sigma = \sqrt{\frac{\Sigma v^2}{n-1}} \qquad \ldots(15.1)$$

where *n* is the number of observations.

If observations have a standard deviation of σ then we can expect that approximately

$\frac{2}{3}$ of the observations will lie within the range $-\sigma$ to $+\sigma$ on either side of the mean;

$\frac{19}{20}$ of the observations within the range -2σ to $+2\sigma$; and

$\frac{399}{400}$ of the observations within the range -3σ to $+3\sigma$. This property of the standard deviation, in indicating the expected distribution of the deviations from the mean, is important for the present purposes.

15.4 *Propagation of errors*

The square of the standard deviation, σ^2, is known as the *variance*. It is a useful measure of precision because if two quantities are added, the variance of the sum is found by adding the variances of the two measurements. For example, if a line is measured in two parts a and b which have standard deviations $\sigma_a = 5$ mm and $\sigma_b = 10$ mm, then the variance of the whole line $a + b$ is

$$\sigma^2_{a+b} = \sigma_a^2 + \sigma_b^2$$
$$5^2 + 10^2 = 125 \text{ mm}^2$$

and its standard deviation σ_{a+b} is

$$\sigma_{a+b} = \sqrt{125} = \pm 11 \text{ mm}.$$

This means that if a line is made up of n measurements each with variance σ^2, the variance of the whole line is $n\sigma^2$ and its standard deviation is $\sqrt{n}\,\sigma$. Conversely if a line is measured n times with the variance of each measurement σ^2, then the variance of the *mean* of all n measurements is

$$\frac{\sigma^2}{n}$$

and its standard deviation

$$\frac{\sigma}{\sqrt{n}}$$

15.5 *Methods of specifying accuracy*

The accuracy of surveys can be safeguarded by specifying,

(a) the equipment to be used and the procedures to be followed; or
(b) the maximum values of miscloses of the measurements, for example the angular misclose of a triangle, or the linear and angular miscloses of a traverse; or
(c) the maximum permitted standard deviation of the results.

Combinations and variations of these methods are also possible.

Method (a) is restrictive as it makes no allowance for variations in technique or improvements in instruments. To be effective it requires very detailed documentation to specify the methods fully. It concentrates on the observations rather than the results. It is not recommended.

Testing the misclose against a standard of 1/8 000 of the distance (or some other specified fraction) is a form of method (b). This also has a number of serious disadvantages. It is the precision of the points determined in the survey that is important, but the traverse misclose is an unreliable indicator of this precision. This is referred to in more detail in para 15.9. The shape of the traverse and the number of sides play a significant part in the relationship. These factors also weaken the power of the test in detecting gross errors. The method of application is inconsistent with the actual distribution of errors. If all accidental errors caused miscloses which were distributed equally over a range $-K$ to $+K$ (figure 13) then we would be justified in accepting all traverses within that range. Only a gross error would push the misclose outside the range. Unfortunately, due to the statistical nature of observations, it is not justifiable to use tests on a simple GO/NO GO basis. The actual curve of error distribution has sides with ever decreasing slope and some arbitrary point has to be chosen, beyond which we consider that an error is so unlikely that it must be a mistake. The limit of $\pm 3\sigma$ is often taken. In these circumstances it is essential to check miscloses from a number of traverses measured under similar circumstances, rather than individual miscloses.

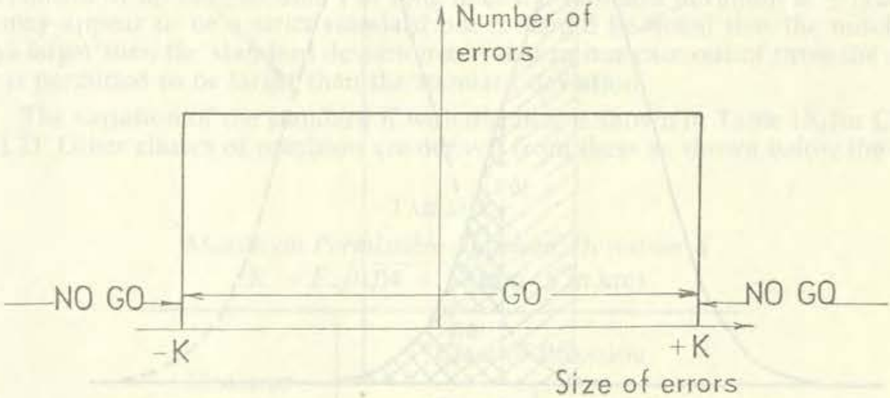


FIGURE 13—FICTITIOUS ERROR DISTRIBUTION

Specifying a misclose as a straight fraction of the length makes it difficult to comply over short distances. One in 8 000 over 8 000 m allows a misclose of one metre, which is easily attained. Over 80 m, it is not so easy to achieve 0.01 m; and over 0.8 m the limit of 0.1 mm is virtually impossible to achieve.

Method (c), specifying the standard deviation, is free of these disadvantages. For integrated surveys a universally applicable standard is required. P. Berthon Jones has proposed an elegant scheme which specifies the maximum standard deviation of points fixed. The next requirement is a method of determining the standard deviation achieved in the actual survey, so that it can be compared with the standard. The same author has described an accurate method. Surveyors may consider the method rather too lengthy for general use. For this reason approximate and simplified forms, which require only a very short calculation, have been devised. These are described under para 15.10 "Simplified Formulae".

15.6 *Detection of gross errors*

There are two aims in specifying accuracy. The first aim is to ensure that the observations and methods applied are sufficiently precise and the second is to detect gross errors. In a sense these two purposes are opposed, because the tolerances are made as wide as possible for economy, but as they are made wider, the power of the test for detecting gross errors is diminished.

If there is a gross error present which has a magnitude of three times the standard deviation, 3σ, the position is as illustrated in figure 14. The actual error distribution is shown, as is the apparent error distribution, displaced 3σ to the right. If all results within the range $\pm 3\sigma$ are accepted then it is clear that 50 per cent of the actual errors will be acceptable. If, however, only results within the $\pm 2\sigma$ range are accepted, only 16 per cent of the actual errors are acceptable. In other words there is an 84 per cent chance that the gross error will be detected. This indicates that the power of detecting gross errors is increased very significantly in this case, by rejecting at 2σ instead of 3σ. However the penalty is that, with the rejection at 2σ, one survey in 20 will have to be repeated even if there are no gross errors present, whereas if 3σ is accepted only one in 400 will need to be repeated. A balance has to be struck between safety and economy.

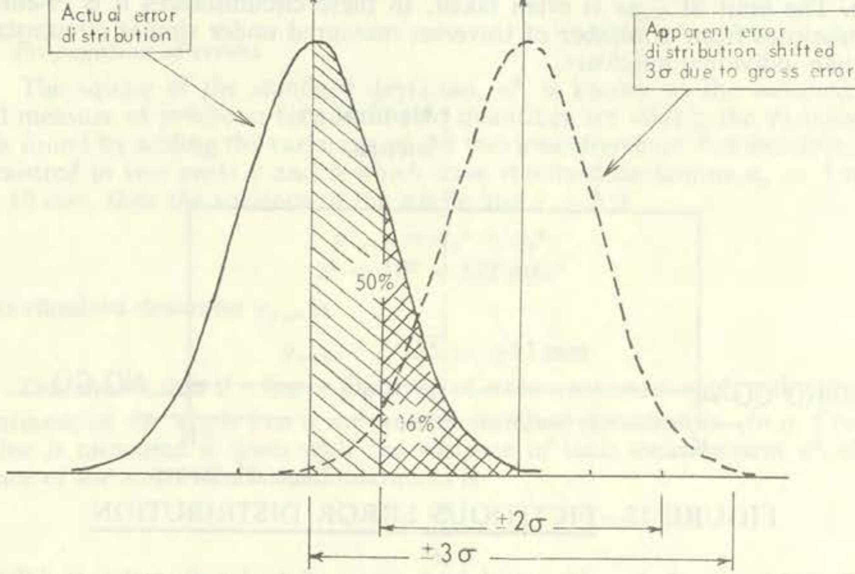


FIGURE 14—DETECTION OF GROSS ERRORS

15.7 *Standards of accuracy*

The standard specified in the Survey Practice Regulations, is that the standard deviation of any point with respect to the control, or with respect to any other point should not exceed

$$K = F\sqrt{0.04 + S^2}\ \text{mm} \qquad \ldots(15.2)$$

where S is the direct distance between the two points, in km, and

F is a constant which is specified for each class of accuracy.

TABLE VIII

Standards of Accuracy

Class of Precision	F	σ for short lines	σ for long lines
A	1.5	0.3 mm	1/667 000
B	5	1.0	1/200 000
C	15	3.0	1/66 700
D	50	10	1/20 000
E	150	30	1/6 670
F	500	100	1/2 000
G	1 500	300	1/667
H	5 000	1 000	1/200

Table VIII shows the value of F appropriate to each class of accuracy. On very short lines, the value of S under the radical in equation (15.2) is negligible and the permissible standard deviation is $F\sqrt{0.04} = \pm 0.2\ F$ mm, while on longer lines the first term becomes negligible and the permissible standard deviation becomes F mm per km or F parts per million. For example control surveys for integration are of Class C precision. The permissible standard deviations are ± 3 mm on short lines and $\pm 1/66\,700$ on long lines.

Class D precision is specified for boundary surveys, for which the appropriate value of F is 50. On very short lines the standard deviation is ± 10 mm with occasional discrepancies of up to ± 20 mm. For long lines the standard deviation is $\pm 1/20\,000$. This may appear to be a strict standard but it should be noted that the misclose is always larger than the standard deviation and that in one case out of three the actual error is permitted to be larger than the standard deviation.

The variation of the standard K with distance is shown in Table IX for Classes C and D. Other classes of precision are derived from these as shown below the table.

TABLE IX

Maximum Permissible Standard Deviation K

$K = F\sqrt{0.04 + S^2}$ mm (S in km)

Distance	Class of Precision	
	C	D
m	mm	mm
0	3.0	10.0
100	3.4	11.2
200	4.2	14
300	5.4	18
400	6.7	22
500	8.1	27
1 000	15	51
2 000	30	100
3 000	45	150
4 000	60	200
5 000	75	250

For Class A, K is $\frac{1}{10}$ of the Class C value
B, K is $\frac{1}{10}$ of the Class D value
E, K is 10 times the Class C value
F, K is 10 times the Class D value
G, K is 100 times the Class C value
H, K is 100 times the Class D value

15.8 *Precision of a traverse*

An important property of least squares adjustments of surveys is that they provide information on the precision of the results. Most surveyors do not yet have access to a computer and least square adjustment programmes, so there is a requirement for less elaborate techniques. Berthon Jones has given formulae for the precision of the weakest point in the traverse, the mid-point, and the standard deviation of the misclose, using as input the co-ordinates of the traverse points X, Y and the standard deviations of the angle and distance measurements. Also required are the co-ordinates X', Y' of the *related traverse*, which coincides with the basic traverse as far as the halfway point H, but thereafter has the direction of each side reversed. In figure 15 for example, A—G is the traverse, D is the halfway point, and from D onwards the direction of the sides are reversed, giving the points E', F', and G'. The related traverse is $A\,B\,C\,D\,E'\,F'\,G'$. The precisions are calculated upon the assumption that the misclose has been adjusted by the Bowditch Rule. It has been shown that the results from a least squares adjustment and a Bowditch adjustment are not significantly different. In the formulae only very approximate co-ordinates are required for substitution. This is important as it enables the traverse to be tested before any observations are made.

The formulae for a traverse of n points, or $m = (n - 1)$ sides are as follows:

For the misclose,

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad \bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \qquad \ldots(15.3)$$

$$I^{YY} = \sum_{i=1}^{n} X_i^2 - n\bar{X}^2 \qquad I^{XX} = \sum_{i=1}^{n} Y_i^2 - n\bar{Y}^2 \qquad \ldots(15.4)$$

$$J = I^{XX} + I^{YY} \qquad \ldots(15.5)$$

$$\sigma_M{}^2 = J\,\sigma_\theta{}^2 + \sum_{i=1}^{n-1} \sigma_{s_i}^2 \qquad \ldots(15.6)$$

where X_i, Y_i are the co-ordinates of the points in the traverse. For a closed traverse, co-ordinates of the starting point are repeated at the end.

σ_θ is the standard deviation of each of the n measured angles (in radians).

σ_{s_i} is the standard deviation of each of the $(n - 1)$ measured lengths.

$\sigma_M{}^2$ is the calculated standard deviation of the misclose which can be compared with the misclose actually achieved.

For the standard deviation of the halfway point H, equations (15.3), (15.4) and (15.5) apply, with co-ordinates X_i, Y_i replaced by the co-ordinates X'_i, Y'_i of the related traverse. Equation (15.5) then yields J', which is used in (15.7) below.

$$\sigma_H{}^2 = \frac{1}{4}\left(J'\,\sigma_\theta{}^2 + \sum_{i=1}^{n-1} \sigma_{s_i}^2\right) \qquad \ldots(15.7)$$

Here $\sigma_H{}^2$ is the total variance of H which is the sum of the variances in any two directions at right angles. A comparison of σ_H should be made with K, the standard set by equation (15.2), substituting the distance to the mid-point for S. Note that in general the mid point has the highest standard deviation of any point in the traverse, and so it is sufficient to test only this point.

Example I

Calculation of the standard deviation σ_M of the misclose and total standard deviation σ_H of the halfway point, of the traverse illustrated in figure 15. *D* is 250 m from one end and 220 from the other and so is chosen as the halfway point. The standard deviation σ_θ of each angle is 10″ and the standard deviation σ_s of each side is ± 5 mm.

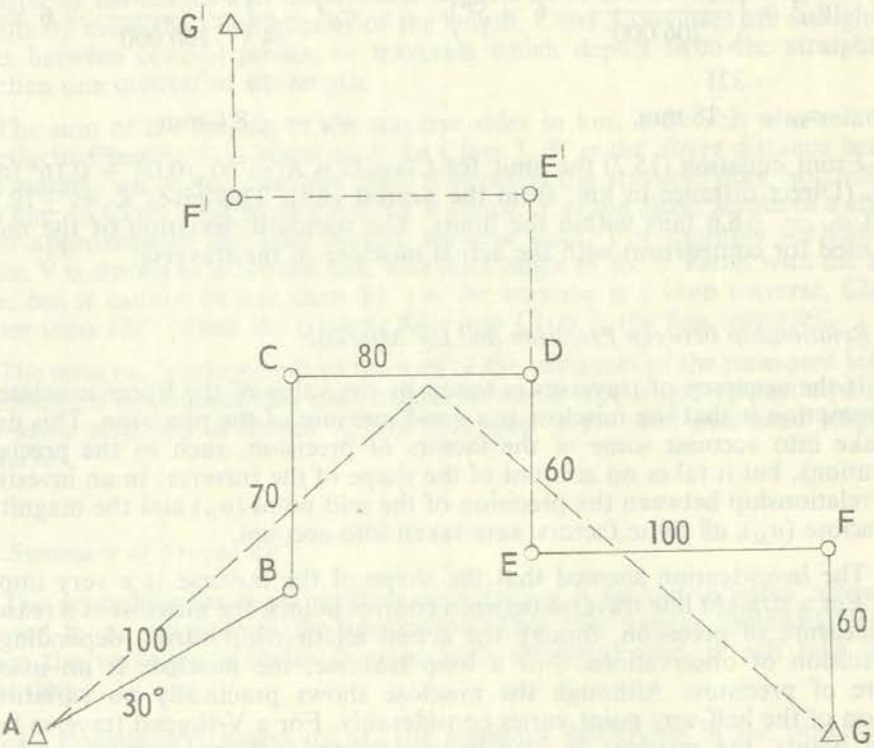


FIGURE 15—TRAVERSE: A B C D E F G n = 7
RELATED TRAVERSE: A B C D E′F′G′

NOTE: Check G D and G′ must lie on a straight line.

Pt	Y	X	Y'	X'
A	0	0	0	0
B	87	50	87	50
C	87	120	87	120
D	167	120	167	120
E	167	60	167	180
F	267	60	67	180
G	267	0	67	240
	1 042	410	642	890
	$\bar{Y}$ 149	$\bar{X}$ 58.6	$\bar{Y}'$ 91.7	$\bar{X}'$ 127

From equation (15.4):

$I^{XX} = 58\,200$ $\qquad I^{X'X'} = 21\,000$

$I^{YY} = 14\,500$ $\qquad I^{Y'Y'} = 40\,800$

From (15.5):

$J = 72\,700\ \text{m}^2 \qquad J' = 61\,800\ \text{m}^2$

$= 7.27 \times 10^{10}\ \text{mm}^2 \qquad = 6.18 \times 10^{10}\ \text{mm}^2$

From (15.6): From (15.7):

$$\sigma_M{}^2 = \left(\frac{7.27 \times 10^{12}}{206\,000} + 6 \times 5^2\right) \qquad \sigma_H{}^2 = \frac{1}{4}\left(\frac{6.18 \times 10^{12}}{206\,000} + 6 \times 5^2\right)$$

$$= 321 \qquad = 74$$

$$\sigma_M = \pm 18\ \text{mm} \qquad \sigma_H = \pm 8.6\ \text{mm}$$

From equation (15.2) the limit for Class D is $K = 50\sqrt{0.04 + 0.16^2}$ for $S = 160$ m. (Direct distance in km, from the nearest end). This gives $K = \pm 12.8$ mm, so that $\sigma_H = \pm 8.6$ falls within the limits. The standard deviation of the misclose, σ_M, is used for comparison with the actual misclose of the traverse.

15.9 *Relationship between Precision and the Misclose*

If the accuracy of traverses is tested by the value of the linear misclose, then the assumption is that the misclose is a good measure of the precision. This measure may take into account some of the factors of precision, such as the precision of observations, but it takes no account of the shape of the traverse. In an investigation of the relationship between the precision of the mid point (σ_H) and the magnitude of the misclose (σ_M), all these factors were taken into account.

The investigation showed that the shape of the traverse is a very important factor. For a straight line traverse between control points, the misclose is a reasonably good measure of precision, though the actual relationship varies, depending upon the precision of observations. For a loop traverse, the misclose is an insensitive measure of precision. Although the misclose shows practically no variation, the precision of the half-way point varies considerably. For a V-shaped traverse between control points, the misclose is actually misleading as a measure of precision, A variation in the factors so as to cause a decrease in precision, also causes the misclose to decrease. In this case the misclose gives a false measure of precision.

The investigation also indicated that it was not necessary to take into account an infinite number of traverse shapes. Provided that there is a reasonable balance between the linear and angular precision, and that the lengths of the traverse sides are fairly even, all traverses can be classified into one of three classes. This is the basis of the simplified formulae for calculating σ_M and σ_H which are described in para 15.10.

For traverses where the above provisos do not hold, the full equations (15.3) to (15.7) should be applied, or, in more difficult cases, rigorous least squares techniques.

15.10 *Simplified Formulae*

Simplified formulae for the calculation of σ_M and σ_H are:

$$\sigma_M = \sqrt{p_j\, SS'\sigma_\theta{}^2 + r\,\sigma_S{}^2}\ \text{mm} \qquad \ldots(15.8)$$

$$\sigma_H = \sqrt{q_j\, SS'\sigma_\theta{}^2 + \tfrac{1}{4}\, r\,\sigma_S{}^2}\ \text{mm} \qquad \ldots(15.9)$$

The symbols are defined in table X where the values of the variables p and q are also tabulated. They depend on the number of sides, m, in the traverse, and on the classification of the traverse shape as Class 1, 2 or 3. *Class 1* includes all traverses which close back onto the starting point or onto a point close to it; that is, onto a point less than one quarter of the total traverse length from the starting point. *Class 2* traverses are basically a V or U shape. The end points are separated by more than one quarter of the length and the traverse departs from the straight line joining the end points by more than one quarter of the length. *Class 3* traverses are straight line traverses between control points, or traverses which depart from the straight line by less than one quarter of the length.

The sum of the lengths of the traverse sides in km, is S. S' is also related to the length. In Class 1, S' is equal to S. In Class 3, S' is the *direct* distance between the end points, which may be less than S because the traverse does not follow a straight line. In Class 2 the basic form is a V shape. S' is the direct length of a straight V which approximates the (less direct) traverse path. For example see figure 15 where the V is shown as a broken line. The apex angle of the V varies with the actual traverse, but it cannot be less than 30° (or the traverse is a loop traverse, Class 1) or greater than 120° (when the traverse falls into Class 3, the line traverse).

The term $r\sigma_S^2$ corresponds to the sum of the variances of the measured lengths. For simplicity it is assumed that each measured whole tape length or part tape length has the same standard deviation σ_S and that there are r such measured lengths in the traverse.

15.11 *Summary of Procedure*

The procedure for checking traverse accuracy is set out in detail below, and is followed by an example of the application. This may appear somewhat lengthy, but once the surveyor becomes familiar with the procedure, it will take only a minimal time.

1. Preliminary: assess σ_θ, σ_S the standard deviations of the observations. (*See* section 16).
2. Examine the traverse and classify as Class 1, 2, or 3. Count the number of sides: m.
3. Look up values of p and q in table X.
4. Assess S and S'.
5. Calculate σ_H (equation (15.9)).
6. Compare σ_H with the standard set by equation (15.2). σ_H indicates the precision of the traverse at its weakest point. If σ_H is more than the standard value K, the design of the traverse is unsatisfactory and it must be upgraded by more precise observations or a change in the shape or number of sides of the traverse.
7. Observe the traverse.
8. Check the observations for accuracy by testing the angular misclose which should not exceed $2\frac{1}{2}\sqrt{n}\,\sigma_\theta$. Check the value of σ_θ from the survey, with standard value of σ_θ.

m	CLASS 1		CLASS 2		CLASS 3	
	p_1	q_1	p_2	q_2	p_3	q_3
1	5.1	2.5	8.5	2.1	11.8	1.3
2	3.9	2.5	7.8	2.0	11.8	1.0
3	3.9	2.8	8.5	2.1	13	1.0
4	4.1	3.1	9.4	2.4	15	1.0
5	4.5	3.5	10.5	2.6	17	1.1
6	4.8	3.9	11.6	2.9	18	1.2
7	5.3	4.3	13	3.2	20	1.3
8	5.7	4.7	14	3.5	22	1.4
9	6.2	5.1	15	3.8	24	1.5
10	6.6	5.5	16	4.1	26	1.7
11	7.1	5.9	17	4.4	28	1.8
12	7.6	6.3	19	4.7	30	1.9
13	8.0	6.7	20	5.0	32	2.0
14	8.5	7.1	21	5.3	34	2.1
15	9.0	7.5	22	5.6	36	2.3
16	9.5	8.0	23	5.9	37	2.4
17	9.9	8.4	25	6.2	39	2.5
18	10.4	8.8	26	6.5	41	2.6
19	10.9	9.2	27	6.8	43	2.7
20	11.4	9.6	28	7.1	45	2.9
25	14	11.7	34	8.6	55	3.5
30	16	14	41	10.1	65	4.1
35	18	16	47	11.7	75	4.7
40	21	18	53	13	84	5.3
45	24	20	59	15	94	5.9
50	26	22	65	16	104	6.5

TABLE X

Simplified Computation of Precision of a Traverse

$$\sigma_M = \sqrt{p_j SS' \sigma_\theta^2 + r\sigma_S^2} \text{ mm} \qquad \ldots(15.8)$$

$$\sigma_H = \sqrt{q_j SS' \sigma_\theta^2 + \tfrac{1}{4} r\sigma_S^2} \text{ mm} \qquad \ldots(15.9)$$

σ_θ —standard deviation of each angle in seconds.

σ_S —standard deviation in mm of each unit of length measurement, eg, each tape length or part thereof.

r —No. of units of length measurement.

S —total length of traverse in km.

S' —length of traverse avoiding zig-zags. (*See* text, para 15.10).

p_j, q_j —factors which depend on No. of sides m in traverse, and on Class of traverse.

j —Class No. of traverse shape.

m —No. of sides in traverse ($= n - 1$).

Class 1—all closed traverses and all traverses where the direct diatance between the end points is less than $S/4$.

Class 2—traverses of more or less U or V shape, where the end points are separated by more than $S/4$, and the traverse wanders off the straight line by more than $S/4$.

Class 3—straight line traverses and those which do not wander off the straight line by more than $S/4$.

9. Check that the approximate figures used in calculating σ_H were satisfactory. If not amend σ_H.
10. Calculate σ_M: (equation (15.8)).
11. Calculate the linear misclose M and compare with σ_M. If $M > 2\frac{1}{2}\sigma_M$ the traverse should be rejected. If $M > 2\sigma_M$ it should be very carefully investigated.
12. Bring M into the running comparison for all traverses of the same type: M should be less than σ_M in two cases out of three, and less than $2\sigma_M$ in 19 cases out of 20.

15.12 *Examples*

Example II. This example is set out in detail following the numbered steps of the "Summary of Procedure", para 15.11. The data is the same as for example I, para 15.8, figure 15.

1. Preliminary: $\sigma_\theta = \pm 10''$ $\sigma_S = \pm 5$ mm.
2. Examination shows it is a V-shaped traverse: Class 2. (End points further apart than $S/4$; deviates off line by more than $S/4$). Number of sides: 6.
3. From Table X, $p_2 = 11.6$; $q_2 = 2.9$.
4. Total length $S = 0.47$ km. S' is length of straight V which approximates traverse (shown dashed in Figure 15). $S' = 0.36$ km.
5. Eq. (15.9)

$$\sigma_H = \sqrt{(2.9 \times 0.47 \times 0.36 \times 10^2) + (\tfrac{1}{4} \times 6 \times 5^2)}$$
$$= \sqrt{86.6} = \underline{\pm 9 \text{ mm}}$$

(Compare with ± 8.6 mm, calculated by alternative method, Example I)

6. Eq. (15.2) Class D. $K = 50\sqrt{0.04 + 0.16^2}$

   $S = 160$ m, direct distance from D to nearest end point.

   $K = \pm 12.8$ mm.

   $\sigma_H < K$. Traverse design is satisfactory.
7. Observe the traverse.
8. Calculate angular misclose of traverse. The standard deviation of this misclose is:

   $$\sqrt{n}\,\sigma_\theta = \sqrt{6} \times 10'' = \pm 24''$$

   If the misclose is $> 48''$, angles should be carefully investigated.

   If $> 60''$, re-observe angles.

   Also use double measurements of angles to verify whether they are consistent with $\sigma_\theta = \pm 10''$.
9. Check. If necessary repeat step 5.
10. $$\sigma_M = \sqrt{(11.6 \times 0.47 \times 0.36 \times 10^2) + (6 \times 5^2)}$$
    $$= \sqrt{346} = \pm 19 \text{ mm}$$

    (Compare with ± 18 mm, calculated in Example I).
11. Compare actual linear misclose M with ± 19 mm.

    If $M > \pm 38$ mm traverse should be investigated.

    If $M > \pm 48$ mm traverse should be rejected.

12. Compare with previous traverses of similar type.
    Is $M < \sigma_M$ in 2 cases out of 3?
    $M < 2\sigma_M$ in 19 cases out of 20?
    If not, the values adopted for σ_θ and σ_S may need revision.

Example III

This example is set out more briefly. It has the same data as Example II in the paper by P. Berthon Jones, *Australian Surveyor*, Vol. 23, No. 7, Sept., 1971, p. 436.

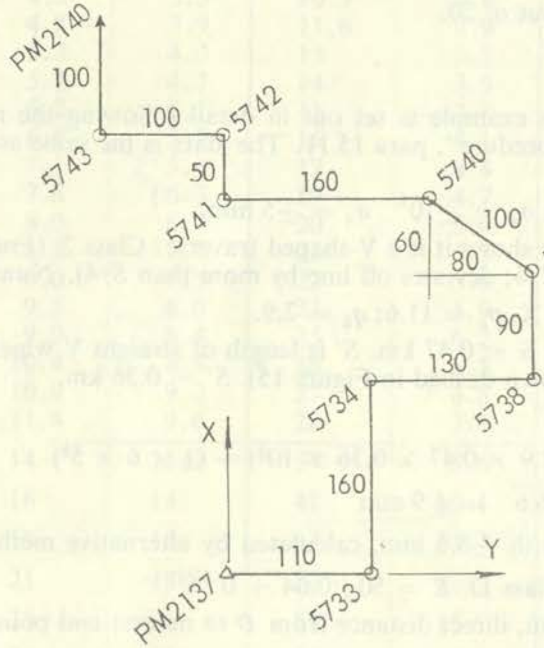


FIGURE 16—TRAVERSE PM2137 TO 5743

Data: See Figure. 16

Class 2 − V shape $\quad m = 9$

$S \;= 1.0 \qquad S' = 0.7$ km

$p_2 = 15 \qquad q_2 = 3.8$ (From Table X)

$$\sigma_M = \sqrt{(15 \times 0.7 \times 72) + (13 \times 5^2)}$$
$$= \pm 33 \text{ mm}$$

$$\sigma_H = \sqrt{(3.8 \times 0.7 \times 72) + (\tfrac{1}{4} \times 13 \times 5^2)}$$
$$= \pm 16 \text{ mm}$$

15.13 *Accuracy of Traverses in Numerical Example*

In the *Numerical Example* of section 12, four traverses are calculated. The accuracy of these traverses is investigated in Examples IV-VII below. For all these examples, σ_θ, the standard deviation of the angle measurements, is taken as $\pm 10''$. The length measurements in Example IV are by edm and the standard deviation of each line is taken as $\pm(5 + 5S)$ mm, where S is the line length in km. For the other examples, where the lengths were measured by steel tape, the standard deviation of each 100 m tape length or part tape length is ± 5 mm.

Example IV Traverse Z—X Section 12, Page *15*

$\sigma_\theta = \pm 10''$

$\sigma_S = \pm(5 + 5S)$ mm $\quad \sigma_{ZP} = \pm 19$

$\sigma_{PQ} = \pm 12$

$\sigma_{QX} = \pm 14$

Second term under root sign, equivalent to $r\sigma_s^2$ becomes the sum of the squares of the standard deviations: $(19^2 + 12^2 + 14^2)$.

The traverse is a straight line traverse—Class 3.

Number of sides $m = 3 \quad p = 13 \quad q = 1.0$ (Table X)

S is the actual length, S' the straight line distance between the ends.

$S = 6.1 \quad S' = 5.4$ km

$\sigma_M = \sqrt{(13 \times 6.1 \times 5.4 \times 10^2) + (19^2 + 12^2 + 14^2)}$

$= \pm 210$ mm

Actual misclose $M = 280$ mm

M is less than twice σ_M, (420 mm) so the traverse measurements pass this test.

$\sigma_H = \sqrt{(1.0 \times 6.1 \times 5.4 \times 10^2) + \frac{1}{4}(19^2 + 12^2 + 14^2)}$

$= \pm 60$ mm

Allowable limit $K = F\sqrt{0.04 + S^2}$

In this case S is the direct distance to the halfway point of the traverse. $S = 2.7$. For a property survey $F = 50$.

$K = 50\sqrt{0.04 + (2.7)^2}$

$= \pm 135$ mm

σ_H is well below K, so the traverse is well within the limit.

Example V Northern Traverse EBAF. Section 12, Page *17* Figure 11, Page *12*

$\sigma_\theta = \pm 10'' \quad \sigma_S = \pm 6$ mm per 100 m tape length or part length.

Number of such lengths: $r = 3 + 13 + 5 + 5 + 1 + 6 + 5 + 4 + 8 = 50$

Traverse is V-shaped, Class 2. Number of sides $m = 9$

$p = 15$; $q = 3.8$ (Table X)

Total traverse length $S = 4.3$

Length of equivalent V traverse $S' = 3.8$

$\sigma_M = \sqrt{(15 \times 4.3 \times 3.8 \times 10^2) + (50 \times 6^2)}$

$= \pm 160$ mm

Actual misclose $M = 170$. Since M is far less than $2\sigma_M (= 320)$ and is in fact barely larger than σ_M, the traverse measurements pass this test.

$\sigma_H = \sqrt{(3.8 \times 4.3 \times 3.8 \times 10^2) + \frac{1}{4}(50 \times 6^2)}$

$= \pm 80$ mm

Allowable limit $K = F\sqrt{0.04 + S^2}$. S is the direct distance to the halfway point, $S = \frac{1}{2}S' = 1.9$

$K = 50\sqrt{0.04 + (1.9)^2}$

$= \pm 95$ mm

σ_H is less than K, so the traverse is acceptable.

Example VI Central Traverse EF. Section 12, Page *18* Figure 10, Page *12.*

$\sigma_\theta = \pm 10''$ $\sigma_S = \pm 6$ mm $r = 1 + 15$

Line traverse, Class 3. $m = 2$

$p = 11.8$; $q = 1.0$ (Table X)

$S = 1.5 = S'$

$$\sigma_M = \sqrt{(11.8 \times (1.5)^2 \times 10^2) + (16 \times 6^2)}$$
$$= \pm 57 \text{ mm}$$

Actual misclose $M = 170$. This is outside the limit $2\sigma_M$ but just within the limit $3\sigma_M$ (= 171). While the traverse measurement need not be rejected automatically, the measurements should be carefully investigated, particularly the standardization of the tape, since the misclose is in the direction of the length of the traverse. If no errors of standardization or computation are located which account for the misclose, it would be advisable to remeasure.

$$\sigma_H = \sqrt{(1.0 \times (1.5)^2 \times 10^2) + \tfrac{1}{4}(16 \times 6^2)}$$
$$= \pm 19 \text{ mm}$$

Allowable limit $K = 50\sqrt{0.04 + S^2}$ $S = 0.7$

$K = \pm 36$ mm

σ_H is less than the allowable limit K (by a wide margin), so that the traverse passes this test.

Example VIII Southern Traverse ECDF. Section 12, Page *18* Figure 10, Page *12*

$\sigma_\theta = \pm 10''$ $\sigma_S = \pm 6$ mm $r = 8 + 17 + 9 + 3 = 37$

Traverse is V-shaped: Class 2. $m = 4$

$p = 9.4$ $q = 2.4$ (Table X)

$S = 3.4$ $S' = 3.0$

$$\sigma_M = \sqrt{(9.4 \times 3.4 \times 3.0 \times 10^2) + (37 \times 6^2)}$$
$$= \pm 105 \text{ mm}$$

Actual misclose $M = 170$ which is within the $2\sigma_M$(= 210) limit. The traverse measurement is acceptable.

$$\sigma_H = \sqrt{(2.4 \times 3.4 \times 3.0 \times 10^2) + \tfrac{1}{4}(37 \times 6^2)}$$
$$= \pm 55 \text{ mm}$$

Allowable limit $K = 50\sqrt{0.04 + S^2}$ $S = 1.2$

$K = \pm 60$ mm

σ_H is less than K, so this traverse is acceptable.

15.14 *General Comments*

In paragraphs 15.1 and 15.11 the various tests of precision are listed. Two of these tests have been applied in the above Examples, IV to VIII. Comparing the actual misclose M against the calculated standard deviation σ_M is a test of the field observations of the traverse. It should tell whether the observations match the expected precision, which has been fed in in the form of σ_θ and σ_S. The comparison is viewed against the probability distribution: in *fourteen* cases out of twenty M should be less than σ_M; in *nineteen* cases out or twenty M should be less than $2\sigma_M$. If M is greater than $2\sigma_M$ it is sufficiently unusual to warrant a thorough investigation, though the measurements need not necessarily be rejected. However if M is greater than $2\frac{1}{2}\sigma_M$ it becomes necessary to reject the measurements and repeat them.

In three of the four examples above, M lies between σ_M and $2\sigma_M$. This is not typical, but four is a small sample and a more typical distribution can be expected after a larger number, say 16 or 20 traverses, have been analysed. If not, then the values adopted for σ_θ and σ_s are suspect and should be investigated by the methods of Section 16.

The standard deviation σ_H of the halfway point of the traverse is calculated in order to compare with the standard deviation allowed by regulation. The comparison tests whether the design of the traverse is satisfactory. If a traverse fails this test, then it is necessary to strengthen it by making the angle or distance measurements more precise, by adding astronomical azimuths or by improving the shape of the traverse. These measures will lead to changes in the values of parameters σ_θ, σ_S and possibly p, q and m used in the calculation. A new calculation can be made to test σ_H. It is obviously better if this test is made before field observations are undertaken.

16. THE STANDARD DEVIATIONS OF OBSERVATIONS

16.1 *Preliminary.*

It is necessary to assess the values of σ_θ and σ_S, the standard deviations of measured angles and measured distances, to enable checks to be made on the accuracy of survey work. Calculations of the standard deviations σ_H and σ_θ in paras 15.8, 15.12 and 15.13 require values of σ_θ and σ_S as input.

Surveyors are constantly aware of the accuracy of their observations, so this requirement is not a new one. Expressing the precision in terms of standard deviations places the assessment on a sound statistical basis and enables them to be used in other assessments.

In the following paragraphs suggestions are made for values of σ_θ and σ_S which may be adopted using various instruments. These are necessarily general and are intended as starting values only. The surveyor is expected to improve and refine them, through analysis of his own observations. Methods for determining σ_θ from field observation are also given. This analysis, and the surveyor's experience, will also indicate the variations in σ_θ and σ_S in varying circumstances and using different equipment.

16.2 *Standard Deviation of Angles, σ_θ*

Table XI indicates values of σ_θ which can be adopted in the absence of more specific data. It is preferable by far to use values of σ_θ calculated by the methods of para 16.4. The tabulated values are based on the assumptions that the centring and the angle observations are carried out with care, and that two arcs (four rounds) or four repetitions are read. Theodolites are divided into two broad categories: 1″ theodolites and 10″ theodolites. The second category includes all those glass-arc theodolites in which the angles are read on a micrometer or estimated to 5″, 10″, or 0.1′ (or even 20″). The effect of eccentricity does not vary as much as might be expected because the experienced surveyor automatically takes greater care in centring when observing over short lines. The values of σ_θ combine the effects of errors of eccentricity of theodolite and target and error in angle measurement.

TABLE XI

Standard deviation of observed angles σ_θ

Theodolite	Distance	σ_θ
10″	70-300 m	±8″
1″	70-300 m	±6″
10″	1000 m	±6″
1″	1000 m	±4″

16.3 *Effects of eccentricity.*

The standard deviation should include the effects of errors from two sources: eccentricity of theodolite and targets, and errors of angle measurements. (In those cases where lateral refraction is significant this source should also be included). The effect of the eccentricities is given approximately by

$$\sigma_e = \frac{400\,e}{\sqrt{ab}} \qquad \ldots(16.1)$$

where e is the standard deviation, in mm, of the centring of the theodolite and the targets sighted, a and b are the lengths of the two sides subtending the angle, and σ_e is given in seconds.

With careful centring e varies between ±1.5 mm (with optical plummet) and ±4 mm (with plumb-bob). Because the surveyor takes more care over centring when lines are short, the angular effect of eccentricity does not vary as rapidly as is suggested by the formula. For example, with lines of 70 m, centring $e = \pm 1.5$. When the lengths reach 300 m, the surveyor relaxes over centring and e might reach 5 or 6 mm. Say $e = \pm 5$. In these two cases:

1. $e = \pm 1.5 \quad \sqrt{ab} = 70 \quad \sigma_e = \pm 8.6''$
2. $e = \pm 5 \quad \sqrt{ab} = 300 \quad \sigma_e = \pm 6.7''$

16.4 *Determination of σ_θ*

Methods 1 and 3 determine a standard deviation σ_a which includes only the effect of angular measurements. It is necessary to add the effects of centring errors σ_e.

$$\sigma_\theta^2 = \sigma_a^2 + \sigma_e^2$$

Method 2 includes both error sources.

Method 1. Where several angles are each observed twice, the differences d between the measures can be used to calculate σ_a.

TABLE XII

Stn	d	d^2
T1	+18″	324
T2	+12	144
T3	−12	144
T4	−12	144
T5	+6	36
T6	+6	36
T7	−12	144
T8	+6	36

$$\Sigma d^2 = 1\,008$$

$$\sigma_d^2 = \frac{\Sigma d^2}{n} = 126$$

$$\therefore \sigma_d = \pm 11.2$$

The standard deviation of each measured angle is $\sigma_d/\sqrt{2}$ but the standard deviation of the mean of both measures of each angle is $\sigma_d/2$.

$$\sigma_a = \pm 5.6''$$

$$\text{If } \sigma_e = \pm 6'' \quad \sigma_\theta = \sqrt{5.6^2 + 6^2}$$

$$= \pm 8''$$

Method 2. The angular miscloses of a number of traverses can be used to compute σ_θ.

TABLE XIII

Traverse	No. of stations n	Misclose t	$\frac{t^2}{n}$
1	10	36	130
2	5	10	20
3	6	24	96
4	8	6	4
5	7	9	12

$$\sum \frac{t^2}{n} = 262$$

$$\sigma_\theta^2 = \frac{1}{5} \sum \frac{t^2}{n} = 52.4$$

$$\sigma_\theta = \pm 7.2''$$

Method 3. An angle may be measured repeatedly, until n measurements have been made. For each measurement, v is the difference from the mean. Then

$$\sigma_a = \sqrt{\frac{\Sigma v^2}{n-1}}$$

In this determination, care needs to be taken to ensure that the measurements are taken under typical field conditions.

Methods 1 and 2 for determining σ_θ require no additional observations, but are merely an analysis of normal field observations. The surveyor can use these to derive his values for σ_θ and later to check whether the results of any particular survey are up to his normal standards. For a reliable value of σ_θ the number, n, of observations used in the determination should be fairly large, say above 12. It is advisable for the surveyor who is applying the methods of this *Manual*, to keep a note book in which he records all his results; all his angular traverse miscloses, for example. At intervals he should calculate a revised value of σ_θ. The results should be separated into categories according to the circumstances of the survey, so that a range of values of σ_θ can be determined for different equipment and varying conditions of surveying.

16.5 *Standard deviation of distance measurement, σ_S*

Values which can be adopted for σ_S, in the absence of more specific data, are, for lines of under one tape length, carefully taped with a steel band,

$$\sigma_S = \pm 6 \text{ mm}$$

and for short range edm

$$\sigma_S = \pm(5 + 5S) \text{ mm}$$

where S is the length in km.

Methods for making determinations of σ_S are given in para 16.6.

In equations (15.8) and (15.9) the term $r\sigma_S{}^2$ is identical with the term $\Sigma\sigma_{S_i}{}^2$ in equations (15.6) and (15.7), being the sum of the squares of the standard deviations of all measured lengths. In the form $r\sigma_S{}^2$ it is assumed that each tape length or part tape length measured has the same deviation σ_S. For example, if measuring with a 100 m band, if the standard deviation σ_S is ± 7 mm per length, the standard deviation of a line of 190 m measured in two sections is

$$\sqrt{r\sigma_S{}^2} = \sqrt{2 \times 7^2} = \pm 10 \text{ mm}$$

If on the other hand it is measured in three sections of 80, 50 and 60 m, the standard deviation

$$\sqrt{r\sigma_S{}^2} = \sqrt{3 \times 7^2} = \pm 12 \text{ mm}$$

The quantity r in (15.8) and (15.9) is the number of units, tape lengths or part tape lengths, in which the total traverse length is measured with σ_S the standard deviation of each unit. For edm it is sufficient to take S as the average length of the traverse lines, and $r\sigma_S{}^2$ becomes $r(5 + 5S)^2$.

16.6 *Determination of σ_S*

Like σ_θ, σ_S is affected by the eccentricities at the end points of the measured line. It is made up of a component due to errors in the length measurement and a component due to eccentricities. For example a line may be measured with a standard deviation $\sigma_e = 5$ mm. If the end points each have eccentricities with a standard deviation $e = 4$ mm then for the measurement:

$$\sigma_S = \sqrt{\sigma_e^2 + e^2 + e^2} = \sqrt{5^2 + 4^2 + 4^2}$$
$$= \pm 7.5 \text{ mm}$$

The standard deviation σ_e of the measurement may be determined by repeated measurement of the same line. Each measurement should, strictly, be made with a different band.

Alternatively σ_S can be determined from the adjustment of a network of traverses, preferably a large network. It is from such a network that the figure $\sigma_S = \pm 6$ mm for carefully taped distances has been calculated, with other evidence to confirm the value.

17. PRECISION OF OTHER SURVEYS

17.1 *Triangulation and mixed control*

Section 15 deals with standards of accuracy in relation to traverse surveys. The assumption is made that the traverses are adjusted by the Bowditch method. Some surveys will be based on other methods, so it is necessary to have methods for determining precision when points are fixed by other techniques. Control surveys and other precise surveys, that is, Class A, B or C surveys will always be computed by least squares methods. Even Class D property surveys might be computed by least squares, if they are extensive or complicated. In this case the computer output should include the elements of the error ellipse for each point. This represents the precision of the point relative to those points which are considered error free, the higher order points, and this is the form required. To derive the standard deviation of a point, take the axes of its error ellipse, square them and add, and take the square root. It is a simple matter to test this standard deviation against the limit which is given by K in equation (15.2).

If points are calculated by semi-graphic methods, then it is more difficult to arrive at a reliable value for the standard deviation, though the error figure gives a clear picture of the magnitude of the errors. If distances v are taken from the final point chosen in the error figure, to each of the n rays in the figure, then the standard deviation is approximately

$$\sqrt{\frac{\Sigma v^2}{n-1}}$$

This is not a reliable value, partly because the number n is always too small. However it is simple, and it is a means of obtaining an estimate.

17.2 *Levelling*

The requirements for precision in levelling are analogous to those for position. The regulations prescribe a maximum value of the standard deviation, depending on the Class of Precision required, and it is necessary for testing, to determine the standard deviation of the actual levelling.

The value for the standard deviation of the elevation between two points is not permitted to exceed C.

$$C = E\sqrt{S} \text{ mm} \qquad \ldots(17.1)$$

S is the distance, in km, between the points and Table XIV gives a value for E, for each class of precision.

TABLE XIV

Classes of Precision in Levelling

Class	E
A	0.3
B	1
C	3
D	10
E	30

If two points are 7 km apart, their elevation difference, measured by Class C levelling, must have a standard deviation less than:

$$C = 3\sqrt{7} = \pm 8 \text{ mm}$$

To test whether the levelling complies with this requirement it is necessary to determine its standard deviation. This depends on distance and is propagated as the square root of the distance. The standard deviation over 1 km, σ_L is determined. To find the value over any other distance S, multiply by $\sqrt{S}$. Thus for example if σ_L is 7 mm, (for $S = 1$) then the standard deviation of the elevation difference over 12 km, is

$$\sigma_L\sqrt{S} = 7\sqrt{12} = \pm 24 \text{ mm}$$

It is assumed that the instruments and procedure are unchanged over the 12 km distance.

Two methods are given for calculating σ_L

1. *Standard deviation from forward and back levelling*

This method makes use of the normal procedure of dividing levelling lines into sections between bench marks, with each section levelled both forward and back. For each section the quantity d^2/b is taken, where d is the difference between the forward and back levelling results, and b is the length of the section in km. Then if there are n such sections:

$$\sigma_L = \sqrt{\frac{\sum \frac{d^2}{b}}{4n}} \qquad \ldots(17.2)$$

This is the standard deviation of the mean of the forward and back elevation differences.

2. *Circuit Closures*

If a number of closed circuits, say n, are levelled, the quantity e^2/c is formed for each circuit, where e is the circuit closure, and c the length of the circuit.

$$\sigma_L = \sqrt{\frac{\sum \frac{e^2}{c}}{n}} \qquad \ldots(17.3)$$

For a reliable value of σ_L, n should be at least 12, preferably around 20. Different values of σ_L will have to be determined for different types of levelling equipment, different procedures or significant differences in other circumstances.

When σ_L is known for a particular levelling operation, it can be tested against the requirement. For a line of length S, the requirement is

$$\sigma_L \sqrt{S} < C$$

i.e. $$\sigma_L \sqrt{S} < E \sqrt{S}$$

From which it may be seen that the requirement is simplified to:

$$\sigma_L < E \qquad \ldots(17.4)$$

PART 5 DETERMINATION OF AZIMUTH FROM SUN OR STAR OBSERVATIONS

18. DETERMINATION OF AZIMUTH

18.1 *Introduction*

Astronomical determinations of azimuth are applicable in certain surveys, generally where state control survey marks have not yet been placed and surveyed or where they are not available to the density required. Surveys where astronomical azimuths are required are listed and discussed in para 6.5. They include isolated surveys which are to be connected later to the Integrated Survey Grid, surveys based on only one control point and long traverse surveys with numerous measured angles.

In this section various methods for determining astronomical azimuth are discussed and the procedure for observation and computation are set out.

18.2 *Conventions*

There are many advantages in using a set of generalised sign conventions in field astronomy and such a system has been adopted in this *Manual*. For a full explanation of the generalized conventions, refer to the *Empire Survey Review*, vol. XII, No. 90, Oct., 1953 and No. 94, Oct., 1954. As we will be only dealing with the southern astronomical triangle for our work the appropriate triangles are shown in fig. 17.

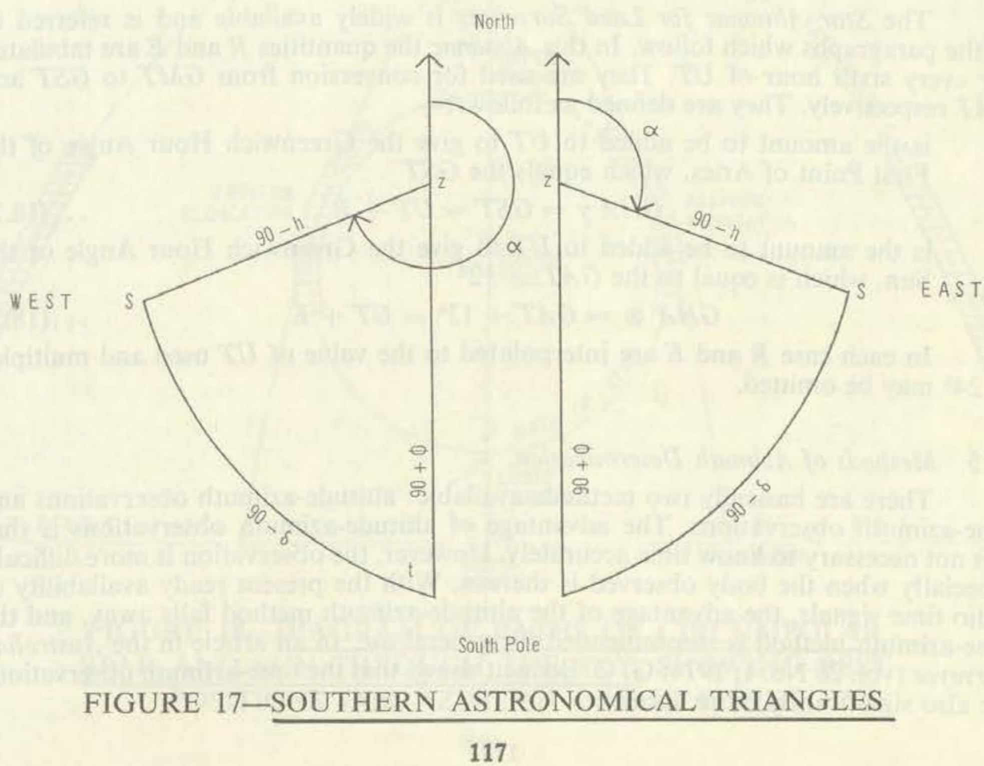


FIGURE 17—SOUTHERN ASTRONOMICAL TRIANGLES

18.3 *Notation*

The following notation is used:

α —azimuth
ϕ —latitude, negative South
λ —longitude, positive East
δ —declination, positive North, negative South
RA —Right Ascension
h —altitude
t —hour angle, always positive
UT —Universal Time = GMT (Greenwich Mean Time)
GST —Greenwich Sidereal Time
GAT —Greenwich Apparent Time
LST —Local Sidereal Time
LAT —Local Apparent Time
ZST —Zone Signal Time
ZT —Zone (Mean) Time
EST —Eastern Standard Time*
$= ZT$ for Zone 10^h East of Greenwich
$$= GMT + 10^h = UT + 10^h \qquad \ldots(18.1)$$

*In the summer months in N.S.W. a time zone of 11^h East of Greenwich is used.

18.4 *Use of the Star Almanac for Surveyors*

The *Star Almanac for Land Surveyors* is widely available and is referred to in the paragraphs which follow. In this *Almanac* the quantities R and E are tabulated for every sixth hour of UT. They are used for conversion from GMT to GST and GAT respectively. They are defined as follows:—

R is the amount to be added to UT to give the Greenwich Hour Angle of the First Point of Aries, which equals the GST

$$GHA\ \gamma = GST = UT + R \qquad \ldots(18.2)$$

E is the amount to be added to UT to give the Greenwich Hour Angle of the Sun, which is equal to the $GAT + 12^h$

$$GHA\ \odot = GAT + 12^h = UT + E \qquad \ldots(18.3)$$

In each case R and E are interpolated to the value of UT used and multiples of 24^h may be omitted.

18.5 *Methods of Azimuth Determination.*

There are basically two methods available: altitude-azimuth observations and time-azimuth observations. The advantage of altitude-azimuth observations is that it is not necessary to know time accurately. However, the observation is more difficult, especially when the body observed is the sun. With the present ready availability of radio time signals, the advantage of the altitude-azimuth method falls away, and the time-azimuth method is recommended for general use. In an article in the *Australian Surveyor* (vol. 26 No. 1, 1974) G. G. Bennett shows that the time-azimuth observations are also significantly more accurate.

In the sections which follow, sections 19 and 20, examples are given of the calculation of time-azimuth observations to a star and to the sun; and, similarly, of the calculation of altitude-azimuth observations to a star and to the sun.

In this *Manual* general formulae are used for the calculations. Special formulae can be used for the special case of circum-elongation azimuth observations to shorten the computation. However this special method is no longer of importance, now that computations can be performed with electronic calculators and for this reason it is omitted from the *Manual*. It is fully covered in the standard text books on field astronomy.

19. TIME-AZIMUTH OBSERVATIONS

19.1 *Arrangement of observations.*

A bright star placed exactly at the celestial pole would be the ideal mark for azimuth observations as its rate of change of azimuth with time would be zero. In the absence of such a star, observations are made onto stars with small rates of change of azimuth with time. For star observations this entails choosing stars as close as possible to the pole, that is, with high southern declinations, and observing them when they are as far as possible from the meridian. The sun is observed for azimuth as soon as possible after sunrise and shortly before sunset (*see* figure 18).

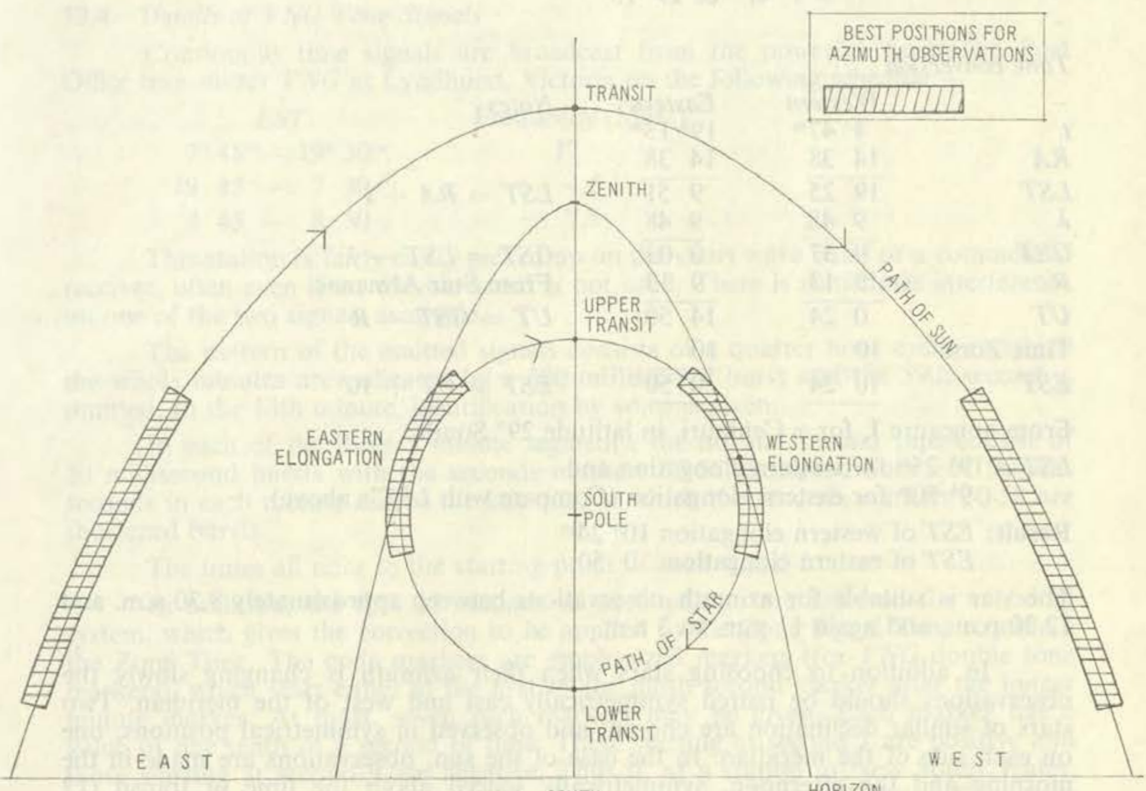


FIGURE 18—DIAGRAM OF PATHS OF CIRCUMPOLAR STAR AND SUN THROUGH THE SKY, SHOWING BEST POSITIONS FOR AZIMUTH OBSERVATIONS.

However if there is good control of time and the longitude of the station is reliable, then results of observations near noon should also be quite reliable.

If one looks towards the South Pole then southern circumpolar stars appear to rotate clockwise, moving westwards over the pole and eastwards under it. The times when they change direction are called eastern and western *elongation.* Since the rate of change of azimuth is zero at these instants, observations for azimuth are favourable at or near elongation; say within 2 hours of elongation, but depending on the declination. The times of elongation of the brighter southern stars are given in graphical form in annexure I. Alternatively the time of elongation can be calculated using the formula:

$$\cos t = \cot \delta \tan \phi \qquad \ldots(19.1)$$

Example: Calculation of times of elongation.

Data: Star α Centauri (RA $14^h\ 38^m$, δ 60° 43′ South) to be observed from station A (ϕ 29° 02′ South, λ $9^h\ 48^m$ East) on 9th February, 1972.

Required: *EST* of eastern and western elongation, calculated to nearest minute.

Hour angle: $\cos t = \cot \delta \tan \phi$

$t = 71°\ 51'$ or $289°\ 09'$

$= 4^h\ 47^m$ or $19^h\ 13^m$

Time conversion

	Western	*Eastern*	*Notes*
t	$4^h\ 47^m$	$19^h\ 13^m$	
RA	14 38	14 38	
LST	19 25	9 51	$LST = RA + t$
λ	9 48	9 48	
GST	9 37	0 03	$GST = LST - \lambda$
R	9 13	9 13	From Star Almanac
UT	0 24	14 50	$UT = GST - R$
Time Zone	10	10	
EST	10 24	0 50	$EST = UT + 10^h$

From annexure I, for α Centauri, in latitude 29° South

LST is $19^h\ 25^m$ for western elongation and

$9^h\ 50^m$ for eastern elongation (Compare with *LST*'s above)

Result: *EST* of western elongation $10^h\ 24^m$

EST of eastern elongation 0 50

The star is suitable for azimuth observations between approximately 8.30 a.m. and 12.30 p.m., and again 11 p.m. to 3 a.m.

In addition to choosing stars when their azimuth is changing slowly the observations should be paired symmetrically east and west of the meridian. Two stars of similar declination are chosen, and observed in symmetrical positions, one on each side of the meridian. In the case of the sun, observations are made in the morning and the afternoon, symmetrically spaced about the time of transit (12 hours *LAT*). This symmetry of observations ensures that, in the mean of azimuths determined from east and west observations the effects of some constant errors are eliminated. Such errors include the error in the latitude adopted for calculation but not the error in determining the clock correction nor the error in the value of longitude used in the computation.

19.2 *Advantages of Time-Azimuth Method*

This method has the following advantages over the altitude-azimuth. The results are independent of vertical refraction and the refraction correction is not required. The observing is easier because attention can be directed to observing only on the vertical cross hair. The vertical circle bubble does not need to be trimmed. The parallax correction for sun observations is not required.

The method is more accurate. However it is necessary to determine the clock correction by comparing it with standard radio or PMG time signals, and to read the clock at the instant of observation. A good quality wrist watch, with a sweep second hand, is a satisfactory "clock".

19.3 *Timing Precision*

The precision to which the time of an observation should be observed depends on the rate at which azimuth is changing. Generally, times are observed to one fifth or one tenth of a second. However, when the star observed is a pole star such as Sigma Octantis, or when any star is observed in the vicinity of elongation, observations to whole seconds of time are quite sufficient. The precision to which the clock correction is determined should be consistent with the precision of the star observations.

19.4 *Details of VNG Time Signals*

Continuous time signals are broadcast from the powerful Australian Post Office transmitter *VNG* at Lyndhurst, Victoria on the following schedule:—

EST	Frequency (MHz)
$7^h\,45^m$— $19^h\,30^m$	12
19 45 — 7 30	4.5
8 45 — 8 30	7.5

This station is fairly easily picked up on the short wave band of a commercial receiver, often even if an outside aerial is not used. There is sometimes interference on one of the two signals available.

The pattern of the emitted signals consists of a quarter hour cycle, in which the whole minutes are indicated by a 500 millisecond burst and the 59th second is omitted. In the 14th minute, identification by voice is given.

In each of the three 5 minute segments, the normal second pips consist of 50 millisecond bursts with the seconds markers 55-58 shortened bursts of 5 milliseconds in each minute except the last one, in which the seconds markers 50-58 are shortened bursts.

The times all refer to the starting point of each signal.

In addition, the first 15 seconds of each minute are reserved for a coding system, which gives the correction to be applied to the Zone Signal Time to obtain the Zone Time. The code markers are emphasized markers (for *VNG* double tone markers), which start either at the first or the ninth second marker after the longer minute marker. At most, seven such markers may be given. Each marker has a value of one tenth of a second of time. These starting at second 1 are positive and those starting at second 9 are negative. Thus if, at a station in New South Wales, 4 emphasized markers are heard with the first one at second 1 then $EST = ZST + 0.4^s$.

19.5 *Observations*

The procedure for determining azimuth from star observations is as follows: The programme begins with a comparison of the clock against radio time signals in order to determine the clock correction on *EST*. A typical set of observations to the reference mark (*RM*) and the star comprises:

1. Horizontal circle reading to *RM*
2. Horizontal circle reading and time to star
3. Repeat 2.
4. Repeat 1.

Four such sets, two on each face, should be taken on a star, followed by four similar sets on the other star of the pair. Finally a further determination of the clock correction should be made.

Observing with a one-second theodolite, the result from a pair of stars observed in the manner described, should approach a precision of 2 arcsecs. If a lesser accuracy is required appropriate modifications can be made to the programme.

Since the star observations are generally at an altitude of 30° or higher, it is important to level the theodolite with great care and to check the levelling between sets.

It is an advantage to prepare a field-book with suitable columns and headings before going into the field.

For sun observations the procedure is similar, though the paired observations are in the morning and the afternoon, rather than to two stars. Because of the longer time gap, the clock correction must be determined before and after the morning observations and the afternoon observations. The readings to the sun should be made tangential to the left limb and the right limb of the sun, alternately. The result will not normally be as precise as star observations. (*see* Bennett, *G. G. Australian Surveyor*, vol. 26, No. 1, 1974, in which precisions of ±6 arcsecs are indicated).

19.6 *Calculation of Azimuth from Time-Azimuth Observations*

The general relationship, which can always be used for computing azimuth from timed observations, is given by

$$\tan \alpha = \frac{-\sin t}{\cos \phi \tan \delta - \sin \phi \cos t} \qquad \text{..(19.2)}$$

for which there is no ambiguity in the quadrant in which α lies provided cognisance is taken of the ratio of the signs of the numerator and denominator. The "TO POLAR" feature of the most small calculators will automatically take this into account.

If the observing period is short (say, less than 10 minutes) calculations can be made from the means of the observations. If not then observations should be calculated separately and the resulting values meaned.

19.7 *Example—Time-Azimuth to Star*

The star α Centauri (*RA* $14^h\ 37^m\ 43.5^s$, δ 60° 43′ 04″ South) was observed to the west of the meridian at a station A (φ 29° 02′ 13″ South λ $9^h\ 47^m\ 31.5^s$ East) on Wednesday, 9th February, 1972. Horizontal circle readings were also taken to Reference Mark B.

Location: Tego, Instrument: T2			Observer: R. Smith		Notes
Watch Time	$10^h12^m19^s$	$10^h19^m02^s$	$10^h31^m01^s$	$10^h36^m53^s$	
Watch Corr'n	+2.0	+3.1	+5.1	+6.1	
EST	10 12 21.0	10 19 05.1	10 31 06.1	10 36 59.1	
Zone	−10	−10	−10	−10	
UT	0 12 21.0	0 19 05.1	0 31 06.1	0 36 59.1	Eq (18.1)
R at 0^h	9 12 47.1	9 12 47.1	9 12 47.1	9 12 47.1	From L.S. Almanac
G S T	9 25 08.1	9 31 52.2	9 43 53.2	9 49 46.2	
λ	9 47 31.5	9 47 31.5	9 47 31.5	9 47 31.5	Eq (18.2)
L S T	19 12 39.6	19 19 23.7	19 31 24.7	19 37 17.7	*LST* =
R A	14 37 43.5	14 37 43.5	14 37 43.5	14 37 43.5	*GST* + λ
t	4 34 56.1	4 41 40.2	4 53 41.2	4 59 34.2	*t*=*LST*−*RA*
t arc	68°44′00″	70°25′00″	73°25′15″	74°53′30″	
φ	−29 02 13	−29 02 13	−29 02 13	−29 02 13	
δ	−60 43 04	−60 43 04	−60 43 04	−60 43 04	
−sin *t*	−0.931 902	−0.942 155	−0.958 426	−0.965 435	
cos *t*	0.362 709	0.335 178	0.285 340	0.260 645	
sin φ	−0.485 373	−0.485 373	−0.485 373	−0.485 373	
cos φ	0.874 307	0.874 307	0.874 307	0.874 307	
tan δ	−1.783 275	−1.783 275	−1.783 275	−1.783 275	
Denom.	−1.383 081	−1.396 444	−1.420 634	−1.432 620	
tan α	0.673 787	0.674 682	0.674 647	0.673 895	Eq (19.2)
α	213°58′18″	214° 00′25″	214°00′20″	213°58′33″	
Obs'd angle *RM* to Star	301 01 43	301 03 47	301 03 53	301 02 04	
Azimuth *RM*	272 56 35	272 56 38	272 56 27	272 56 29	

The mean value of the azimuth to *B* is 272°56′32″

19.8 *Example, Time-Azimuth to Sun.*

The following is a determination from sun observations made on the morning and afternoon of 10th January, 1968, at a station ADC (ϕ 33° 51′ 53″ South, λ 10^h 04^m 48.6^s East). The reference mark used was station P.

Location: Sydney	Instrument: T2	Observer: R. Smith	Notes
EST	$7^h42^m40.0^s$	$16^h19^m02.5^s$	
Zone	−10	−10	
UT	21 42 40.0	6 19 02.5	Eq (18.1)
E	11 52 57.4	11 52 48.5	From L.S. Almanac
λ	10 04 48.6	10 04 48.6	
t	19 40 26.0	4 16 39.6	$t = UT + E + \lambda$
t arc	295°06′30″	64°09′54″	
ϕ	−33 51 53	−33 51 53	
δ	−22 08 22	−22 05 19	
−sin *t*	0.905 507	−0.900 053	
cos *t*	0.424 331	0.435 780	
sin ϕ	−0.557 234	−0.557 234	
cos ϕ	0.830 356	0.830 356	
tan δ	−0.406 860	−0.405 826	
Denom	−0.101 387	−0.094 148	
tan α	+8.931 213	+9.559 957	Eq (19.2)
α	96°23′19″	264°01′42″	
Obs'd angle sun to *RM*	269 00 20	101 22 01	
Azimuth *RM*	5 23 39	5 23 43	
	Observed mean azimuth of *P* is 5°23′41″		

20. ALTITUDE-AZIMUTH OBSERVATIONS

20.1 *Arrangement of observations*

These observations should be made when the change in altitude is large compared with the change in azimuth. This indicates that these observations should also be made around the time of elongation, and well away from the time of transit. (*See* figure 18). If the observations are paired symmetrically east and west of the meridian, constant errors in the observed altitudes and the adopted latitude will be eliminated in the mean of the azimuths derived. To achieve this with sun observations, early morning and late afternoon observations should be made but there is no guarantee that any systematic errors in the refraction correction will be of the same size and sign in both morning and afternoon observations.

The observing procedure is as in time-azimuth observations, except that the vertical circle reading to the celestial body replaces the accurate time reading. The zone time at which the observations are made should be noted to the nearest minute so that the sun's declination, which is tabulated in the *Star Almanac for Land Surveyors* with respect to *UT*, can be interpolated to sufficient acuracy. The operation of setting a star onto the intersection of the crosshairs is not difficult. When observing the sun's disc, it has to be set so that it is tangential to both the horizontal and vertical crosshairs. A difficulty arises because the points of tangency are separated by a considerable distance in the field of view and it is impossible to see both simultaneously. Observations should be paired in opposite quadrants of the field of view.

If the observing period is short, calculations can be made using the means of the altitudes and horizontal circle readings. If not, then calculations should be made using individual observations and taking the mean of the results.

20.2 *Calculation of azimuth from altitude-azimuth observations*

The azimuth α can be calculated from the rigorous general relationship.

$$\cos \alpha = \frac{\sin \delta - \sin h \sin \phi}{\cos h \cos \phi} \qquad \ldots(20.1)$$

The ambiguity in placing α in its correct quadrant may be resolved from consideration of the following table:—

	cos α +	cos α −
East Obs'vn	$Q1$	$Q2$
West Obs'vn	$Q4$	$Q3$

The observed altitudes must be corrected for refraction. This requires readings of the air temperature and pressure at the time of observation. The correction is in the form:

$$\text{Refraction} = r_0 f \qquad \ldots(20.2)$$

where r_0 and f are tabulated in the Almanac. Refraction is *subtracted* from observed altitudes.

In addition altitudes observed to the sun should be corrected for parallax which is given by

$$\text{Parallax} = 9'' \cos h \qquad \ldots(20.3)$$

Parallax is *added* to observed altitudes.

20.3 *Example. Altitude-Azimuth to Star*

The star α Centauri (*RA* $14^h37^m43.5^s$, δ 60°43′04″ South) was observed to the west of the meridian at a station A(φ 29°02′13″ South, λ $9^h47^m31.5^s$ East) on Wednesday, 9th February, 1972. The reference mark observed was *B*.

Location: Tego	Instrument: T2		Observer: R. Smith		Notes
EST	10^h12^m	10^h19^m	10^h31^m	10^h37^m	
Zone	−10	−10	−10	−10	
UT	0 12	0 19	0 31	0 37	Eq (18.1)
Barometer	1 000 mb	1 000 mb	1 000 mb	1 000 mb	
Thermometer	30°C	30°C	30°C	30°C	
f	0.92	0.92	0.92	0.92	From L.S.
r_0	82″	84″	89″	92″	Alamnac
Obs'd altitude	35°21′44″	34°32′27″	33°04′20″	32°21′18″	
Refraction	−1 15	−1 17	−1 22	−1 25	Eq (20.2)
Parallax	—	—	—	—	(star)
h	35°20′29″	34°31′10″	33°02′58″	32°19′53″	
φ	−29 02 13	−29 02 13	−29 02 13	−29 02 13	
δ	−60 43 04	−60 43 04	−60 43 04	−60 43 04	
sin h	0.578 447	0.556 686	0.545 363	0.534 815	
cos h	0.815 720	0.823 934	0.838 200	0.844 969	
sin δ	−0.872 221	−0.872 221	−0.872 221	−0.872 221	
sin φ	−0.485 373	−0.485 373	−0.485 373	−0.485 373	
cos φ	0.874 307	0.874 307	0.874 307	0.874 307	
sin h sin φ	−0.280 763	−0.275 054	−0.264 704	−0.259 585	
sin δ − sin h sin φ	−0.591 458	−0.597 167	−0.607 517	−0.612 636	
cos h cos φ	0.713 190	0.720 371	0.732 844	0.738 762	
cos α	−0.829 313	−0.828 971	−0.828 985	−0.829 273	Eq (20.1)
α (West Obs'n)	213°58′18″	214°00′25″	214°00′20″	213°58′33″	
Obs'd angle *RM* to star	301 01 43	301 03 47	301 03 53	301 02 04	
Azimuth *RM*	272 56 35	272 56 38	272 56 27	272 56 29	

The mean value of the azimuth to *B* is 272°56′32″

20.4 *Example. Altitude—azimuth to the sun.*

An example of the computation of azimuth from morning and afternoon observations of the sun at similar altitudes is tabulated below. Observations were made on the morning and afternoon of 10th January, 1968 at a station *ADC* (ϕ 33°51′53″ S, λ 10^h05^m E) using the reference mark *P*.

Location: Sydney	Instrument: T2	Observer: R. Smith	Notes
EST	7^h43^m	16^h19^m	
Zone	−10	−10	
UT	21 43 (9.1.68)	6 19 (10.1.68)	Eq (18.1)
Barometer	1005 mb	1009 mb	
Thermometer	20°C	24°C	
f	0.96	0.95	From L.S. Almanac
r_0	91	89	From L.S. Almanac
Obs'd altitude	32°27′32″	33°02′06″	
Refraction	−1 27	−1 24	Eq (20.2)
Parallax	+ 7	+ 7	Eq (20.3)
h	32°26′12″	33°00′49″	
ϕ	−33 51 53	−33 51 53	
δ	−22 08 22	−22 05 19	
$\sin h$	0.536 367	0.544 838	
$\cos h$	0.843 985	0.838 541	
$\sin \delta$	−0.376 862	−0.376 040	
$\sin \phi$	−0.557 234	−0.557 234	
$\cos \phi$	0.830 356	0.830 356	
$\sin h \sin \phi$	−0.298 882	−0.303 603	
$\sin \delta - \sin h \sin \phi$	−0.077 980	−0.072 437	
$\cos h \cos \phi$	0.700 808	0.696 287	
$\cos \alpha$	−0.111 272	−0.104 034	Eq (20.1)
α	96°23′19″ (E Obs)	264°01′42″ (W Obs)	
Obs'd angle Sun to *RM*	269 00 20	101 22 01	
Azimuth *RM*	5 23 39	5 23 43	

The mean value of the azimuth to *P* is 5°23′41″

PART 6 THE AUSTRALIAN HEIGHT DATUM

21. A NEW DATUM FOR LEVELLING IN NEW SOUTH WALES

21.1 *Introduction*

On 5th May, 1971, the Division of National Mapping, on behalf of the National Mapping Council of Australia, completed the simultaneous adjustment of a network of first and third-order levelling embracing the entire Australian continent, holding mean sea level fixed at zero at thirty tide-gauges round the mainland coast. The resulting datum surface has been termed the Australian Height Datum, A.H.D. 1971, and has been adopted as the datum for all mapping activities by the National Mapping Council. It is the purpose of the following paragraphs to describe the establishment of the National Levelling Survey and the methods of computation and adjustment.

Bench marks in New South Wales have for many years been referred to Standard Datum, which is the value of mean sea level defined by the tide gauge at Fort Denison in 1897. The reasons why it has been considered desirable to change the long-established State Datum are set out, and the resolutions agreed to between State and Commonwealth authorities for the future conduct of levelling within the State of New South Wales are stated.

21.2 *The New South Wales State Standard Datum*

The first move to establish a common origin for levelling bench marks in New South Wales was made in 1897. Mean Sea Level was recommended for adoption as the Standard Datum for levelling throughout the Colony. The only tidal records available at this time were those which were kept at Fort Denison (Sydney Harbour) and at Newcastle. The former records were adopted. The value of mean sea level at Fort Denison, as defined by the tide gauge there, was computed by the Government Astronomer from a series of readings extending over 13 years. To make the Datum for levels more readily accessible to surveyors, it was necessary to connect some conveniently located mark on the mainland to the gauge, which was separated therefrom by 604 m of water. The brass plug let into the Bridge Street wall of the Lands Department building was adopted for this purpose. Four separate sets of levelling were combined, and the level of the brass plug was found to be 8.821 m above mean sea level. This value was known as "Standard Datum" and remained unchallenged until the advent of the "Australian Height Datum" (A.H.D.) in 1971.

21.3 *Tide-gauges on the New South Wales Coast*

The 1897 Conference recommended that additional tide gauges be established along the coast, for determining mean sea level and mean high water, to provide information for engineering purposes, shipping, and for the determination of legal boundaries of land fronting tidal waters. Tide gauges have thus been established at various times in the intervening period at Tweed Heads, Evans Head, Ballina, Iluka (Clarence River), Yamba, Coffs Harbour, Crowdy Head, Port Stephens, Newcastle, Camp Cove (near the entrance to Port Jackson) Port Kembla, Jervis Bay, Bermagui, Moruya and Eden.

The tidal gauge at Camp Cove commenced readings about 1916. Although the Fort Denison Gauge had remained the standard for the port of Sydney, the one at Camp Cove is regarded as more suitably located to record true ocean conditions.

21.4 *First-order Levelling in the Sydney Metropolitan Area*

A network of precise levelling was extended over the Sydney metropolitan area by the Central Mapping Authority, Department of Lands in 1953-7. An adjustment of the network was carried out to give provisional reduced levels of bench marks in 1960. The reduced levels obtained above were based on the 1897 value of the Lands Department plug. These levels were not published, but have been used for all subsequent levelling operations.

21.5 *Levelling in Country Areas of New South Wales*

Commencing in 1954, and continuing to the present time, a network of 1st order levelling has been extended over the Eastern and part of the Central Divisions of New South Wales. In 1961, the Commonwealth Government made funds available for 3rd order levelling surveys. The levelling was done by contract under the supervision and control of the Surveyor General. The 1st order levelling along the coast and tablelands was undertaken by the Central Mapping Authority (*see* annexure J). As the 1st and 3rd order levelling was gradually extended over the State, reduced levels were made available as the sections were completed. The values were necessarily provisional, being subject to adjustment for the closure of loops, and to further re-adjustment when the whole network was completed. The datum for these levels was the 1897 value for the Lands Department plug.

Reduced levels for bench marks in country areas were first published in 1964 for work completed to that date. By 1965-6, it became feasible to effect an adjustment of the entire State network, and these results were published in 1968. These levels were still based on the 1897 value for the Lands Department plug. Connections were made to a number of tide gauges, but no attempt was made to reconcile differences that were revealed at these locations.

21.6 *The Australian Levelling Survey*

At the inception of the National Mapping Council in 1945 it had been agreed that the Director of National Mapping would co-ordinate the activities of Commonwealth and State authorities in the mapping of Australia, subject to the Council's recommendations. Within this charter, the Director planned and organized the 3rd order levelling betewen 1961 and 1966. The Division of National Mapping undertook the collection of levelling and tide gauge data and the necessary analysis and processing of the data, culminating in the adjustment of a single homogenous network of levelling covering the whole of mainland Australia.

In 1964, the Council resolved to encourage responsible authorities to install tide gauges to obtain simultaneous observations for mean sea level on a national basis. In 1967, a survey of twenty-six tide gauges situated on the mainland coast of Australia was undertaken by officers of the Division of National Mapping, with the aim of calibrating the gauges, connecting them to bench marks and discussing their operation with the operators.

The tide gauges visited by the Survey party on the New South Wales Coast were those at Coffs Harbour, Camp Cove (Sydney Harbour) and Eden. The latter was eventually excluded at the request of the Surveyors-General of New South Wales and Victoria because of the unsatisfactory readings obtained, and an additional gauge at Port Kembla was surveyed by officers of the New South Wales Department of Lands.

The epoch for all of the gauges (except that at Karumba in the Gulf of Carpentaria) was 1st January, 1966, to 31st December, 1968.

21.7 *The National Levelling Adjustment*

Details of the method of adjustment used for this large network of levelling are given by Roelse, Granger and Graham (*see Bibliography*). The procedure is based on the principle of an adjustment in phases, instead of in one single operation, carrying forward the full variance-covariance matrix from one phase to the next.

After collection of data and attending to special problems at junction points and at state borders, the first stage produced orthometrically corrected height differences between bench marks.

The total number of junction points was too great for the adjustment to be effected in one operation. Consequently, the network had to be divided into a series of regions, roughly corresponding to State borders, and a preliminary adjustment made of each. The data for the New South Wales region formed one of these preliminary adjustments with an origin at Sydney.

After the preliminary adjustment every junction point in each of the five adopted regional networks was connected to its own local origin and, accordingly, any junction points on the boundary common to two regions were connected to both origins. These derived height differences, which joined these common junction points to two regional origins were now used to form condition equations.

The second phase of the adjustment reduced the separate results of the regional adjustments into a consistent whole. The adjustment was carried out twice, first as a free adjustment, with only one height fixed, and secondly with the height of mean sea level at each tide gauge fixed at zero. This second adjustment established the Australian Height Datum.

Finally the heights between junction points were adjusted in a linear manner. Supplementary levelling, not included in the main adjustment, was adjusted into the A.H.D. similarly.

Summarizing, the Australian Height Datum (A.H.D.) is the datum surface derived from a simultaneous adjustment of the Australian continental levelling network, including thirty tide gauges held fixed at the values of mean sea level. The observations for mean sea level are for the epoch 1st January, 1966 to 31st December, 1968, except at Karumba, where the epoch was 1st January, 1957, to 31st December, 1960. (*See* annexure J).

21.8 *The Sydney Metropolitan Area*

The following table shows a comparison of the heights of four bench marks on the perimeter of the Sydney Metropolitan Levelling Network based on State Standard Datum and the Australian Height Datum, with the corresponding values of the Lands Department Plug and the Camp Cove tide-gauge.

Bench Mark	*Australian Height Datum (A.H.D.)*	*Standard Datum (S.D.)*	*Differences (S.D.)—(A.H.D.)*
	metres	*metres*	*metres*
Heathcote (PM1938)	194.228	194.276	+0.048
Hornsby (PM225)	201.050	201.091	+0.041
Parramatta (PM222)	9.279	9.330	+0.051
Liverpool (PM646)	20.075	20.132	+0.057
Lands Dept plug	8.775	8.821	+0.046
Zero, Camp Cove Tide-gauge	−0.927	−0.887	+0.040

Because of the large amount of levelling already in existence in Sydney metropolitan area, it was at first considered essential that the long established heights adopted by State authorities for bench marks should be retained at their existing values.

A committee of delegates from various government departments and instrumentalities, the universities, and the Institution of Surveyors, Aust., N.S.W. Division, was convened by the Surveyor-General to consider the proposals submitted by the Director of National Mapping to deal with this problem.

At the initial meeting the committee considered that the established and proclaimed level of the Camp Cove tide gauge (0.887 m) should be adopted for the National adjustment instead of the value derived in the National Tide Gauge Survey for 1966-8 (0.927 m) in view of the large number of established points and consequent levelling. This, in effect, meant that by the adoption of the 1897 value for mean sea level at Fort Denison, the existing heights for bench marks in the Metropolitan area would remain unchanged.

However, on further investigation, this would have involved the adoption of an obviously incorrect datum and one inconsistent with the others in the adopted epoch. In addition to this the suggested "platform" proved to be dished and the resulting "buffer" zone would introduce a gradient of 0.058 m between Liverpool (fringe of the platform) and Port Kembla (the first tide gauge south of Sydney) a distance of only 79 km. This was obviously not acceptable as a basis for a National and State datum and the committee, without exception, agreed that the new datum (A.H.D.) should replace the existing standard datum. The new datum was to be introduced entirely in metres with all existing levelling being re-adjusted, converted and introduced as soon as possible.

From the table above it will be seen that the maximum difference in the height occurs at Liverpool, where it amounts to 0.057 metres, whereas at the Lands Department plug it is 0.046 metres. This discrepancy of 0.011 metres is within the specifications for 1st order levelling, so that for practical purposes, if levels on State Standard Datum in the Metropolitan area are required at any future time, it will be sufficient to apply a correction of +0.046 to the corresponding A.H.D. value in metres.

As a check on the value of mean sea level at Camp Cove, for the epoch 1966-8, which defines the Australian Height Datum in the Sydney Metropolitan area, the mean tide level readings for the tide gauge at Camp Cove for the ideal 19-year cycle 1951-69 were computed. The result was 0.933 metres, which agrees closely with the value for mean sea level (0.927 metres) derived by the Horace Lamb Institute for 1966-8.

It is of interest to compare the corresponding values at the Fort Denison gauge for the same epoch. A new connection between the Lands Department plug and the zero of the tide-gauge was effected in 1940 by Messrs Hart and Doyle (Maritime Services Board and Public Works Department, respectively) who obtained a difference in height of 9.698 metres, and a further connection was made by the Central Mapping Authority, Department of Lands, in 1953, when a difference of 9.700 metres was obtained.

	metres
Difference in height between zero of Fort Denison gauge and plug in north wall of Department of Lands, Bridge Street, Sydney by 1953 levelling	9.700
Height of plug of Australian Height Datum	8.775
Mean Sea Level (A.H.D.) transferred to the Fort Denison gauge	0.925
Mean Tide level on Fort Denison gauge for period 1951-69	0.936
Difference	0.011

For the original 1897 height difference between the plug and zero on the gauge (9.716 m) the agreement is even closer. Fort Denison is within Port Jackson about 6 km from the open sea whereas Camp Cove is immediately adjacent to the Heads. The two factors of possible tidal gradient within the harbour and the difficulty of transferring the gauge level from the Island at Fort Denison to the mainland at Bennelong Point or Mrs Macquarie's Point, over about 600 m, throw further doubt on the suitability of Fort Denison for a major datum point.

21.9 *The adoption of the Australian Height Datum in New South Wales*

In view of the variability of tidal records over the years in the Sydney region when compared with those used to derive the previously adopted value for mean sea level, and the desirability of introducing a reliable level datum which may be expected to remain unchanged, the committee decided that the Australian Height Datum should be adopted throughout the State. The adoption of the metric system of units gave added impetus for the decision. No objection to alteration of Standard Datum can be envisaged on legal grounds provided that the difference between the datums may be ascertained when required.

Following discussions between representatives of the Central Mapping Authority, Department of Lands and of the Division of National Mapping, the following five recommendations, agreeable to both parties, were submitted as the basis of future levelling operations in the State.

Resolutions

(1) All Bench marks in New South Wales are to be referred to the Australian Height Datum.

(2) In the event that sections of levelling already adjusted on the A.H.D. by the Division of National Mapping are re-run either wholly or in part the responsibility of re-adjusting such sections lies with that Division. The responsibility for determination of values of bench marks along new levelling sections connected to already adjusted levelling rests with the Surveyor General. Heights of previously determined junction points and other fixed points shall not be disturbed without the concurrence of the Director of National Mapping.

(3) Newly determined heights are to be notified to the Director of National Mapping as soon as possible.

(4) Where new and additional first order levelling is run between traverses already adjusted to the Australian Height Datum and agreement between this first order levelling and existing values on the Australian Height Datum is to a first order standard of accuracy, simple linear adjustment can be effected.

(5) Should first order standards not be attained as in (4), such cases are to be discussed and dealt with separately.

22. DETERMINATION OF MEAN HIGH WATER MARK

22.1 *Lands Survey Directions on Determination of Mean High Water Mark*

The New South Wales Department of Lands Survey Directions on "Determination of Mean High Water Mark" have been reviewed and the amended A.H.D. version is set out in the following paragraphs.

22.2 *Legal Significance*

Mean high water (M.H.W.) is by Common Law, or by Statute in the case of land below mean high water mark which is vested in the Maritime Services Board, the boundary of all land having frontage to tidal water. It is defined as the mean of all high tides (including both spring and neap tides) taken over a long period. Tidal waters may be either salt or fresh, and embrace all waters in coastal streams to the tidal limit. The tidal limit is often indicated on Department of Lands plans of survey but its actual location at any time should be defined by observation.

22.3 *Consent of Crown to Definition.*

It is necessary to obtain the consent of the responsible Crown authority to the fixation of any boundary which defines mean high water. The responsible authority for the control of all land below mean high water in Sydney Harbour, Botany Bay and Port Hunter is the Maritime Services Board. In all other cases, except where the land below mean high water mark is specifically vested in another authority, (e.g., Public Works Department) or is held in fee simple, the Department of Lands is the responsible authority. In cases where the consent of the Department of Lands to the definition is necessary, it is the practice to not require a fresh consent where a determination has been agreed to within the previous 10 years and where no substantial variation in position has occurred since the previous determination.

22.4 *Methods of Determination—Preliminary*

The determination of the limit of mean high water presents no difficulty when the foreshore is steep. On flat grades and where mangrove swamps exist, great care is necessary.

The zero of a number of tide gauges at which Mean High Waters have been determined over a long period have been connected to the Australian Height Datum (A.H.D.). The location of these gauges and the relation between A.H.D. and M.H.W. are recorded in the schedule hereunder.

	Mean Tidal Range	Mean Tide above Gauge Zero	M.H.W. above Gauge Zero	A.H.D. above Gauge Zero	M.H.W.M. R.L. A.H.D.
Camp Cove	1.058	0.933	1.462	0.927	0.535
Fort Denison	1.067	0.936	1.469	0.925	0.544
Newcastle	1.041	0.966	1.487	1.010	0.477
Port Kembla	0.973	0.881	1.367	0.872	0.495
Coffs Harbour	1.055	0.808	1.335	0.821	0.514

Note: The tide values shown above for Camp Cove, Fort Denison, Newcastle, Port Kembla, and Coffs Harbour are taken from records for the 19 year period 1951 to 1969.

Information relating to other tide gauges located along the coast of New South Wales is not at present available for publication. However as the information comes available it will be released.

22.5 *Description of Methods.*

1. *Levelling from Bench Mark.*

In locations on the sea coast and in close proximity of the gauges shown in the schedule in para 22.4 above, the position of mean high water can be fixed by normal differential levelling procedures from bench marks related to A.H.D. This method cannot be used with accuracy in positions within estuaries and streams unless reliable information on tidal gradients is available. Tidal gradients vary with the shape of an estuary and distances from the open sea, and while generally there is a raising in level of the mean tidal plane as the distance upstream increases, this does not apply to the various elements such as mean high water, mean low water etc. As an example, in Sydney Harbour, mean high water level at the head of the Parramatta River is 0.08 m higher than at Fort Denison while at Homebush Bay it is 0.01 m lower than at Fort Denison.

2. *Levelling from a Local Tide Gauge.*

There are so many influences tending to disturb the rhythmical flow of the tide that observations extending over at least 12 months are necessary to obtain accurate results. A much simpler process to give the approximate required level is to take the mean of all the high waters observed at a tide gauge located at the site over a full lunation period of 29 days. Care must be taken to obtain the height of all the night tides as well as the day tides. This is necessary because of the fact that in summer the day tides are higher, and in winter the night tides are higher, the inequality becoming greater as the moon's declination, either north or south, increases. In spring and autumn there is very little difference. Factors which tend to increase the height of the tide are:—

(a) The moon with southern declination;

(b) The moon in perigee;

(c) Long continual southerly winds; and

(d) A very low barometer.

Any combination of the above or of the opposite conditions should be avoided as far as possible. This method will give an approximation of the height of M.H.W. to ± 0.015 metres. Therefore, on a very flat foreshore, the mean of several lunation periods should be observed. Where a local temporary tide gauge is constructed for this purpose, it should be so positioned that it is protected from wave action and wind conditions which could vary the water surface level. Flood or fresh conditions should, of course, be avoided.

3. *The Range Ratio Method.*

In a stream or estuary where an automatic tide recorder is in operation and long-term values of the various tidal planes exist, a fairly accurate value of mean high water may be obtained by observation on one day of the high and low water levels at the place where the determination of high water mark is required. The range ratio method consists of observing the level of high and low water for consecutive tides at that place, calculating the mean of these levels, and applying a formula to calculate the level of mean high water at that place using tidal information ascertained at the location of the automatic recorder.

The calculations from the observations made at that place and the information obtained relating to the automatic recorder will give a difference in level to be applied to observed high water level to obtain the value of mean hig water. The formula is set out hereunder:—

fixed by normal differential levelling procedures from bench marks related to A.H.D. This method cannot be used with accuracy in positions within estuaries and streams, unless reliable information on tidal gradients is available. Tidal gradients vary with the shape of an estuary and distances from the open sea, and while generally there is a raising in level of the mean tidal plane as the distance upstream increases, this does not apply to the various elements such as mean high water, mean low water etc. As an example, in Sydney Harbour, mean high water level at the head of the Parramatta River is 0.08 m higher than at Fort Denison while at Homebush Bay it is 0.01 m lower than at Fort Denison.

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Any combination of the above or of the opposite conditions should be avoided as far as possible. This method will give an approximation of the height of M.H.W. to ±0.015 metres. Therefore, on a very flat foreshore, the mean of several lunation periods should be observed. Where a local temporary tide gauge is constructed for this purpose, it should be so positioned that it is protected from wave action and wind conditions which could vary the water surface level. Flood or fresh conditions should, of course, be avoided.

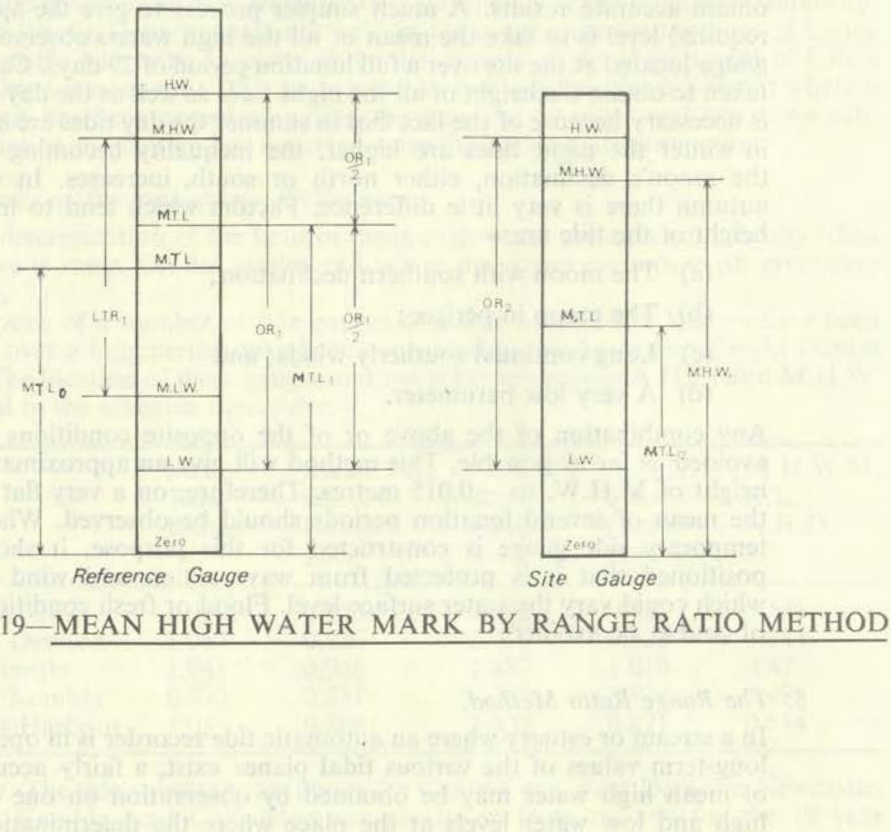


FIGURE 19—MEAN HIGH WATER MARK BY RANGE RATIO METHOD

3. *The Range Ratio Method.*

In a stream or estuary where an automatic tide recorder is in operation and long-term values of the various tidal planes exist, a fairly accurate value of mean high water may be obtained by observation on one day of the high and low water levels at the place where the determination of high water mark is required. The range ratio method consists of observing the level of high and low water for consecutive tides at that place, calculating the mean of these levels, and applying a formula to calculate the level of mean high water at that place using tidal information ascertained at the location of the automatic recorder.

The calculations from the observations made at that place and the information obtained relating to the automatic recorder will give a difference in level to be applied to observed high water level to obtain the value of mean high water. The formula is set out hereunder:—

$$\text{Approx. M.H.W.} = \text{M.T.L.}_{\cdot 2} + K_1 + K_2$$

where

$$K_1 = \text{M.T.L.}_{\cdot 0} - \text{M.T.L.}_{\cdot 1}$$

$$K_2 = \frac{\text{L.T.R.} \times \text{O.R.}_{\cdot 2}}{2 \times \text{O.R.}_{\cdot 1}}$$

$\text{M.T.L.}_{\cdot 2}$ = Mean of observed high water and low water at site gauge (M.T.L. is Mean Tide Level)

$\text{M.T.L.}_{\cdot 0}$ = Long-term value on the gauge of mean tide level at automatic gauge. (See schedule, para 22.4).

$\text{M.T.L.}_{\cdot 1}$ = Mean of observed high water and low water at automatic gauge. (Obtainable from controlling authority).

L.T.R. = Difference between Mean high water and Mean low water at automatic gauge. (Mean Range—*See* schedule para 22.4).

$\text{O.R.}_{\cdot 2}$ = Observed range at site gauge.

$\text{O.R.}_{\cdot 1}$ = Observed range at automatic gauge. (Obtainable from controlling authority).

These elements are diagrammatically illustrated in Figure 19 attached. The observed range should be determined from consecutive tides during a period of spring tides occurring at the time of new moon and full moon not affected by abnormal weather conditions or floods. When using this method prior arrangements should be made with the authority controlling the gauge to obtain the required information on the day of observation.

22.6 *Further information.*

For technical assistance or advice on tidal gradient information and the limit of tidal influence, enquiry should be directed in regard to Sydney Harbour, Botany Bay and Port Hunter to the Survey Branch, Maritime Services Board, Sydney, and in regard to other estuaries to the Survey Branch, Department of Public Works, Sydney.

GLOSSARY

GLOSSARY

NOTE. This glossary is intended to provide a simple explanation of technical terms to be found in the *Manual*. It should not be regarded as definitive, as the accent is on simplicity rather than comprehensive accuracy. More accurate definitions and fuller explanations will be found in the main sections.

*indicates a reference to another term which appears in the Glossary.

arc-to-chord correction.

See: projection corrections

Australian Height Datum (AHD)

A system of control points for height based on a network of levelling measurements which covered the whole of Australia and which was fitted to mean sea level as measured at tide gauges distributed around the Australian coast, over the period 1968-1970.

Australian Map Grid (AMG)

A rectangular co-ordinate system* drawn on a Transverse Mercator Projection* with zones 6° wide, adopted by the National Mapping Council, for mapping throughout Australia.

Australian National Spheroid (ANS)

A spheroid with particular dimensions and shape. Major semi-axis $a =$ 6 378 160 metres and flattening $f = 1/298.25$. This has been adopted for the computation of geographic positions by the National Mapping Council, and is the standard spheroid for Australia.

azimuth

A direction in the horizontal plane measured clockwise from the true north meridian.

bearing

The bearing of a line is the clockwise angle from the x axis of a rectangular co-ordinate system*, to the line.

central meridian

See: Transverse Mercator Projection

co-ordination

The organisation of surveys carried out by various authorities so that they are co-related, in particular by being connected to a set of base surveys. Its meaning is similar to that of integration but the provisions of the Survey Co-ordination Act did not go as far as is currently envisaged by survey integration.

co-ordinate system

See: rectangular co-ordinate system

geoid

The surface defined by mean sea level over the oceans and the level which would be taken up by the sea surface if it were to extend under the continents. In shape this surface approximates to a spheroid* but, in detail, it has undulations above and below a spheroidal surface.

geodetic survey

A survey covering a large area, so that the effect of the earth's curvature must be taken into account, and carried out to high precision. Often applied to the first-order control survey of a country.

graticule

See: projection

grid

See: projection

grid convergence

See: projection corrections

ground distance

A measured distance to which a correction for slope has been applied, but not corrections for reduction to sea level or to the projection. (*See also:* projection distance)

Integrated Survey Grid (ISG)

A rectangular co-ordinate system* drawn on a Transverse Mercator Projection with zones 2° wide, designed for integrated surveys in New South Wales.

network

The imaginary pattern formed by joining the points of a survey by lines indicating measurements between them. Also indicating the whole of the survey connecting the points. For example, levelling network, control network, trigonometrical survey network.

projection

A representation, on a plane, of the surface of the earth, or, strictly, of the spheroid*. The lines of latitude and longitude on the projection form the *graticule*. In general these lines are curved. The lines representing a system of rectangular co-ordinates on the projection form a *grid*, so called because they form a square grid. Computations on this grid are very much simpler and more convenient than on the spheroid.

projection corrections

The curved surface of the spheroid cannot be represented exactly on a plane. To account for the differences, small corrections must be applied to measured quantities before they can be used in computations on the projection. These corrections are the *scale correction*, to be applied to distances, the *arc-to-chord* or *(t − T) correction* to be applied to bearings* and the *grid convergence* to be applied to azimuths*. For a distance measured above the spheroid or sea level, the sea-level correction* must be applied, although it is not a *projection* correction.

projection distance

The distance between two points on the plane of the projection, which differs from the ground distance* by the amount of the scale and sea-level corrections.

rectangular co-ordinate system

A reference system of a set of parallel lines and a second set at right angles to the first, enabling the position of any point to be expressed uniquely by two *co-ordinates*, *y* and *x* or Easting E and Northing N.

scale correction

See: projection corrections

sea level correction

This correction reduces the distance between points situated above sea level to the distance between the corresponding points on the sea level or spheroid surface. It arises because the verticals from two neighbouring points converge as they are followed downwards to the centre of the earth.

spheroid

The figure described by an ellipse when it is rotated around its minor axis. The dimensions of a spheroid are chosen so as to approximate, as nearly as possible, the geoid* surface. The Australian National Spheroid* has been chosen for use in Australia.

State control survey marks

A comprehensive set of points of known horizontal position and height, established so as to form the starting points and basis of integrated surveys. The points are marked by monuments of standard form. The marks are placed, surveyed and maintained under the supervision of the Surveyor-General who is responsible for maintaining records of the marks and their positions.

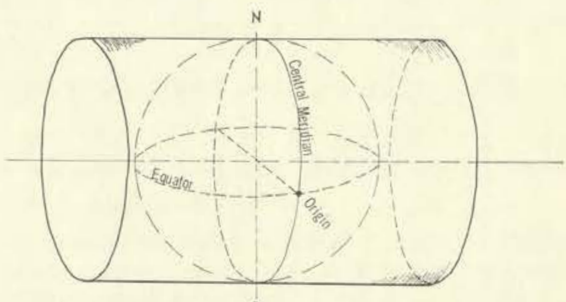
survey integration

A system in which surveys of all types are interrelated by basing them on State survey control marks*, expressing the results in the form, inter alia, of ISG co-ordinates, and retaining records of such surveys for public use.

Transverse Mercator Projection (TM)

In the normal Mercator Projection the spheroid is projected onto a cylinder wrapped around it so as to touch it along the equator. (See Fig 20A). In the Transverse Mercator Projection the cylinder touches the spheroid not around

A



B

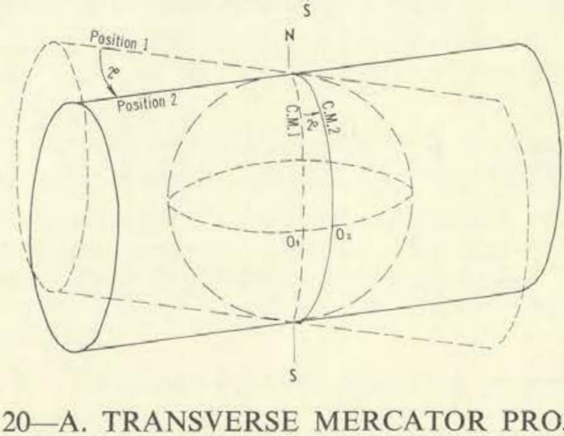


FIGURE 20—A. TRANSVERSE MERCATOR PROJECTION.
B. POSITIONS OF CYLINDERS WITH CENTRAL MERIDIANS 2° APART.

the equator but around a meridian called the *central meridian*. If only a narrow zone centred on this meridian is used, then the projection corrections can be kept very small. In order to cover a large area, it is necessary to move the cylinder around to another central meridian so as to cover a new zone which is adjacent to the first. (Fig 20B). The *zone width* is the width, in degrees, of the zones into which a country is divided. The narrower the zones, the smaller the projection corrections. For the AMG*, the zones, which are 6° wide were chosen for mapping purposes. For integrated surveys, 2° zones have been chosen for the New South Wales ISG*.

trigonometrical survey

A control survey, carried out by precise methods, normally by triangulation. The name refers to the method of computation, using trigonometrical relationships.

zone width

See: Transverse Mercator Projection.

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ANNEXURES

ANNEXURE A.

INTEGRATED SURVEY GRID. N.S.W.
PROJECTION FACTORS.
Accuracy, 1 part in 100 000

SEA LEVEL FACTOR.

Feet	Factor	Metres	Feet	Factor	Metres	Feet	Factor	Metres
0		0	2 194		669	4 493		1 370
	1.00000			0.99989			0.99978	
104		32	2 403		733	4 702		1 433
	0.99999			0.99988			0.99977	
313		96	2 612		796	4 911		1 497
	0.99998			0.99987			0.99976	
522		159	2 821		860	5 120		1 561
	0.99997			0.99986			0.99975	
731		223	3 030		924	5 329		1 624
	0.99996			0.99985			0.99974	
940		287	3 239		987	5 538		1 688
	0.99995			0.99984			0.99973	
1 149		350	3 448		1 051	5 747		1 752
	0.99994			0.99983			0.99972	
1 358		414	3 657		1 115	5 956		1 815
	0.99993			0.99982			0.99971	
1 567		478	3 866		1 178	6 165		1 879
	0.99992			0.99981			0.99970	
1 776		541	4 075		1 242	6 374		1 948
	0.99991			0.99980			0.99969	
1 985		605	4 284		1 306	6 583		2 007
	0.99990			0.99979			0.99968	
2 194		669	4 493		1 370	6 792		2 070

Example:

	In metres	In feet
h	= 780	= 2 559
d	= 4 670.67 (ground distance)	= 15 323.72
Factor	= 0.999 88	= 0.999 88 (from table)
s	= 4 670.67 × 0.999 88	= 15 323.72 × 0.999 88
	= 4 670.11 (spheroidal dist.)	= 15 321.88

SCALE FACTOR (k)

E(km)	k	E(km)	E(km)	k	E(km)
300		300	383		217
	0.99994			1.00003	
320		280	388		212
	0.99995			1.00004	
335		265	392		208
	0.99996			1.00005	
345		255	397		203
	0.99997			1.00006	
353		247	401		199
	0.99998			1.00007	
360		240	405		195
	0.99999			1.00008	
367		233	408		192
	1.00000			1.00009	
373		227	412		188
	1.00001			1.00010	
378		222	416		184
	1.00002			1.00011	
383		217	419		181
				1.00012	
			423		177

Example:

	In metres	In feet
	$E = 398\,000$ m $= 398.0$ km	$= 398\,000$ m $= 398.0$ km
	$s = 4\,670.11$ m	$= 15\,321.88$ ft
	$k = 1.00006$	$= 1.00006$ (from table)
	$S = 4\,670.11 \times 1.00006$	$= 15\,321.88 \times 1.00006$
	$= 4\,670.39$ (Grid distance)	$= 15\,322.80$

ANNEXURE B

INTEGRATED SURVEY GRID. N.S.W.
PROJECTION CORRECTIONS.
SEA LEVEL CORRECTION per 1 000 units

Feet	Corrn.	Metres	Feet	Corrn.	Metres	Feet	Corrn.	Metres
0		0	2 194		669	4 493		1 370
	0			−0.11			−0.22	
104		32	2 403		733	4 702		1 433
	−0.01			−0.12			−0.23	
313		96	2 612		796	4 911		1 497
	−0.02			−0.13			−0.24	
522		159	2 821		860	5 120		1 561
	−0.03			−0.14			−0.25	
731		223	3 030		924	5 329		1 624
	−0.04			−0.15			−0.26	
940		287	3 239		987	5 538		1 688
	−0.05			−0.16			−0.27	
1 149		350	3 448		1 051	5 747		1 752
	−0.06			−0.17			−0.28	
1 358		414	3 657		1 115	5 956		1 815
	−0.07			−0.18			−0.29	
1 567		478	3 866		1 178	6 165		1 879
	−0.08			−0.19			−0.30	
1 776		541	4 075		1 242	6 374		1 943
	−0.09			−0.20			−0.31	
1 985		605	4 284		1 306	6 583		2 007
	−0.10			−0.21			−0.32	
2 194		669	4 493		1 370	6 792		2 070

Example: In metres / In feet

h = 780 = 2 559

d = 4 670.67 (ground dist.) = 15 323.72

$d.10^{-3}$ = 4.67 = 15.32

Correction = −0.12 × 4.67 = −0.12 × 15.32

= −0.56 = −1.84

s = 4 670.67 − 0.56 = 15 323.72 − 1.84

= 4 670.11 (spheroidal dist.) = 15 321.88

SCALE CORRECTION per 1 000 units

E(km)	Corrn.	E(km)	E(km)	Corrn.	E(km)
300		300	383		217
	−0.06			+0.03	
320		280	388		212
	−0.05			+0.04	
335		265	392		208
	−0.04			+0.05	
345		255	397		203
	−0.03			+0.06	
353		247	401		199
	0.02			+0.07	
360		240	405		195
	−0.01			+0.08	
367		233	408		192
	0.00			+0.09	
373		227	412		188
	+0.01			+0.10	
378		222	416		184
	+0.02			+0.11	
383		217	419		181
				+0.12	
			423		177

Example:

	In metres	In feet
E	= 398 000 = 398.0 km	= 398 000 = 398.0 km
s	= 4670.11 (spheroidal dist.)	=15 321.88
Corrn.	= +0.06 (from table)	= +0.06
Proj. corrn.	= +(4.67 × 0.06)	= +(15.32 × 0.06)
	= +0.28	= +0.92
S	= 4 670.11 + 0.28	= 15 321.88 + 0.92
	= 4 670.39	= 15 322.80

INTEGRATED SURVEY GRID, N.S.W.

ARC-TO-CHORD CORRECTION (δ)

Easting (km)

ΔN(km)	300	310	320	330	340	350	360	370	380	390	400	410	420	430
0	0″	0″	0″	0″	0″	0″	0″	0″	0″	0″	0″	0″	0″	0″
5	0	0	0	0	1	1	1	1	1	1	1	1	2	2
10	0	0	1	1	1	1	2	2	2	2	3	3	3	3
15	0	0	1	1	2	2	2	3	3	3	4	4	5	5
20	0	1	1	2	2	3	3	4	4	5	5	6	6	7
25	0	1	1	2	3	3	4	4	5	6	6	7	8	8
30	0	1	2	2	3	4	5	5	6	7	8	8	9	10
35	0	1	2	3	4	4	5	6	7	8	9	10	11	12
40	0	1	2	3	4	5	6	7	8	9	10	11	12	13
45	0	1	2	3	5	6	7	8	9	10	11	13	14	15
50	0	1	3	4	5	6	8	9	10	11	13	14	15	17
	300	290	280	270	260	250	240	230	220	210	200	190	180	170

Easting (km)

Example:

E = 398 000 m
= 398.0 km
ΔN = 10.0 km

The absolute value of δ is 3 seconds from the table.
Obtain the sign from figure 2 , pg 20.

$\delta = -3$ seconds.

GRID CONVERGENCE, $\gamma'' = (E - 300\ 000) \times 10^{-5} \times C$

Table of C

N(km)	0	10	20	30	40	50	60	70	80	90	Differences (negative) 1	2	3	4	5	6	7	8	9
700	2 600	2 592	2 583	2 575	2 567	2 559	2 551	2 542	2 534	2 526	1	2	2	3	4	5	6	7	7
800	2 518	2 509	2 501	2 493	2 485	2 477	2 469	2 461	2 453	2 445	1	2	2	3	4	5	6	6	7
900	2 437	2 429	2 421	2 413	2 405	2 398	2 390	2 382	2 374	2 366	1	2	2	3	4	5	6	6	7
1 000	2 358	2 351	2 343	2 335	2 327	2 320	2 312	2 304	2 297	2 289	1	2	2	3	4	5	5	6	7
1 100	2 281	2 274	2 266	2 259	2 251	2 244	2 236	2 229	2 221	2 214	1	2	2	3	4	5	5	6	7
1 200	2 206	2 199	2 191	2 184	2 177	2 169	2 162	2 155	2 147	2 140	1	1	2	3	4	4	5	6	7
1 300	2 133	2 125	2 118	2 111	2 104	2 096	2 089	2 082	2 075	2 068	1	1	2	3	4	4	5	6	6
1 400	2 061	2 053	2 046	2 039	2 032	2 025	2 018	2 011	2 004	1 997	1	1	2	3	4	4	5	6	6
1 500	1 990	1 983	1 976	1 969	1 962	1 955	1 948	1 941	1 934	1 927	1	1	2	3	3	4	5	6	6
1 600	1 920	1 913	1 907	1 900	1 893	1 886	1 879	1 872	1 866	1 859	1	1	2	3	3	4	5	5	6
1 700	1 852	1 846	1 839	1 832	1 825	1 819	1 812	1 805	1 799	1 792	1	1	2	3	3	4	5	5	6
1 800	1 785	1 779	1 772	1 765	1 759	1 752	1 746	1 739	1 733	1 726	1	1	2	3	3	4	5	5	6

Example:

$E = 398\ 000$ m
$N = 1\ 255\ 000$ m
$E - 300\ 000 = +98\ 000$ m
$C = 2\ 169 - 4 = 2\ 165$ (from table)

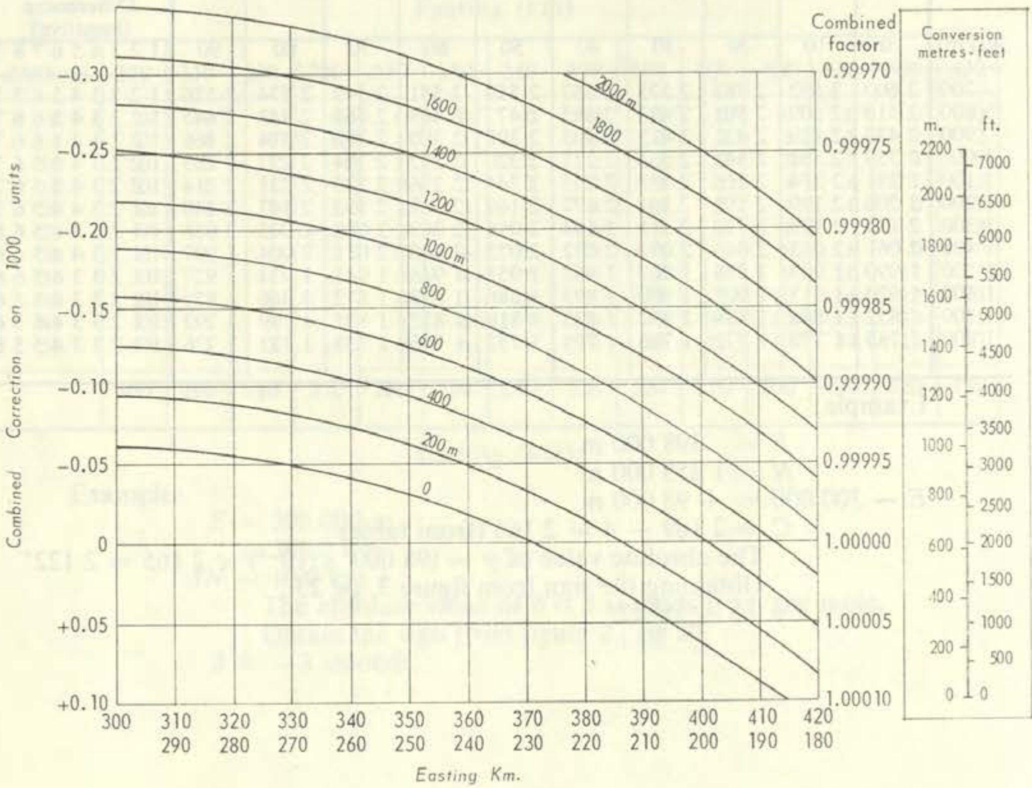
The absolute value of $\gamma = (98\ 000 \times 10^{-5}) \times 2\ 165 = 2\ 122''$

Obtaining the sign from figure 3, pg 23.

$\gamma = +35' 22''$

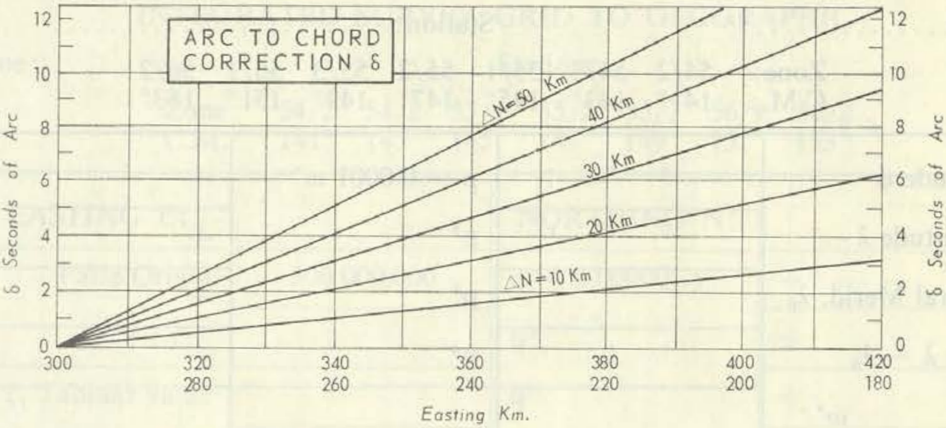
INTEGRATED SURVEY GRID, N.S.W.

COMBINED SCALE AND SEA LEVEL CORRECTIONS



Example: $d = 4\,670.67$ m (ground dist.); $h = 680$ m; $E = 398\,000$ m.
$S = 4\,670.67 - 0.05 \,.\, 4.67 = 4\,670.44$ (grid dist.)

or $S = 4\,670.67 \times 0.99995 = 4\,670.44$



Example: $E = 398\ 000$ m
$\Delta N = 10\ 000$ m $= 10$ km.
$\delta = 3''$
Estimate Easting as noted in 6.2 and obtain sign from figure 2, pg 20.

TRANSFORMATION OF CO-ORDINATES.
GEOGRAPHIC TO INTEGRATED SURVEY GRID

Zone:.............. Station:........................

Zone:	54/2	54/3	55/1	55/2	55/3	56/1	56/2
C.M.:	141°	143°	145°	147°	149°	151°	153°

Latitude ϕ	° ′ ″	$p = 0.0001\ \omega''$	+
Longitude λ	° ′ ″	p^2	+
Central Merid. λ_0	°	p^3	+
$\omega = \lambda - \lambda_0$	° ′ ″	p^4	+
ω''			
a_0 Tabular value		a_1 Tabular value	
Increment		Increment	
a_0 for ϕ		a_1 for ϕ	
a_2 Tabular value		a_3 Tabular value	
Increment		Increment	
a_2 for ϕ		a_3 for ϕ	
a_4 for ϕ		a_1p	+
a_0	+	a_3p^3	+
a_2p^2	—	y	
a_4p^4	—	False Origin	300 000.000
NORTHING N		EASTING E	
b_1 Tabular value			
Increment			
b_1 for ϕ			
b_3			
b_1p	+		
b_3p^3	+		
$\pm\gamma''$			
GRID CONVERGENCE γ			

Formulae.

$$y = a_1\,p + a_3\,p^3$$

$$E = 300\,000 + y$$

$$N = a_0 - a_2\,p^2 - a_4\,p^4$$

$$\gamma = b_1\,p + b_3\,p^3$$

Computed by:................/ /.. Checked by: ..../../..

TRANSFORMATION OF CO-ORDINATES
INTEGRATED SURVEY GRID TO GEOGRAPHIC

Zone:.................... Station:..........................

Zone:	54/2	54/3	55/1	55/2	55/3	56/1	56/2
C.M.:	141°	143°	145°	147°	149°	151°	153°

EASTING E:		NORTHING N:	
False Origin	300 000.000	$q = 0.000001\ y$	+
y±		q^2	+
c_1 Tabular value		q^3	+
Increment		q^4	+
c_1			
c_3 Tabular value		Interp N in a_0 ϕ'	° ′ ″
c_3 Increment		c_2 Tabular value	
c_3		Increment	
$c_1 q$	+ ″	c_2	
$c_3 q^3$	−	c^4	
E_5	+	ϕ'	° ′ ″
$\pm\omega$	″	$c_2.q^2$	− ″
Central Merid. λ_0	°	$c_4.q^4$	+ ″
LONGITUDE λ	° ′ ″	LATITUDE ϕ	° ′ ″
d_1 Tabular value			
Increment			
d_1			
d_3			
$d_1 q$	+ ″		
$d_3 q^3$	− ″		
γ''	″		
GRID CONVERG. γ	° ′ ″		

Formulae

$$\phi = \phi' - c_2\, q^2 + c_4\, q^4$$

$$\omega = c_1\, q - c_3\, q^3 + E_5$$

$$\gamma = d_1\, q - d_3\, q^3$$

$$\lambda = \lambda_0 \pm \omega$$

Computed by:....................../../.. Checked by:................../../..

INTEGRATED SURVEY GRID, N.S.W.
ZONE TO ZONE TRANSFORMATION

$k_1(y)-k_2(x)+k_3=K_3$ $K_3(y)-K_4(x)+k_5=K_5$ $E'=E'_0+\Delta E+K_5(y)-K_6(x)$ $\Delta E=E-E_0$ $(y)=\Delta E.10^{-5}$

$k_1(x)+k_2(y)+k_4=K_4$ $K_3(x)+K_4(y)+k_6=K_6$ $N'=N'_0+\Delta N+K_5(x)-K_6(y)$ $\Delta N=N-N_0$ $(x)=\Delta N.10^{-5}$

ϕ	E'_0 West	E'_0 East	N'_0	k_1	$\lvert k_2\rvert$	$\lvert k_3\rvert$	k_4	k_5	$\lvert k_6\rvert$
28° 00′	398 359.135	201 640.865	1 901 330.441	+ 0.0036	0.0690	24.2623	− 0.5966	− 13.4296	1638.8233
30	397 901.321	202 098.679	1 845 917.131	+ 0.0035	0.0701	24.1467	− 0.6035	− 13.8729	1665.6539
29° 00′	397 436.056	202 563.944	1 790 499.866	+ 0.0034	0.0712	24.0293	− 0.6102	− 14.3214	1692.3575
30	396 963.373	203 036.627	1 735 078.602	+ 0.0032	0.0723	23.9101	− 0.6167	− 14.7747	1718.9321
30° 00′	396 483.309	203 516.691	1 679 653.295	+ 0.0031	0.0734	23.7891	− 0.6230	− 15.2328	1745.3756
30	395 995.896	204 004.104	1 624 223.904	+ 0.0030	0.0744	23.6663	− 0.6291	− 15.6955	1771.6860
31° 00′	395 501.172	204 498.828	1 568 790.385	+ 0.0028	0.0755	23.5416	− 0.6351	− 16.1628	1797.8613
30	394 999.173	205 000.827	1 513 352.701	+ 0.0027	0.0766	23.4152	− 0.6408	− 16.6344	1823.8995
32° 00′	394 489.935	205 510.065	1 457 910.813	+ 0.0026	0.0776	23.2870	− 0.6463	− 17.1102	1849.7987
30	393 973.495	206 026.505	1 402 464.683	+ 0.0024	0.0787	23.1570	− 0.6517	− 17.5901	1875.5569
33° 00′	393 449.892	206 550.108	1 347 014.273	+ 0.0023	0.0797	23.0253	− 0.6568	− 18.0738	1901.1720
30	392 919.164	207 080.836	1 291 559.552	+ 0.0022	0.0808	22.8918	− 0.6618	− 18.5614	1926.6423
34° 00′	392 381.350	207 618.650	1 236 100.484	+ 0.0020	0.0818	22.7565	− 0.6665	− 19.0526	1951.9656
30	391 836.490	208 163.510	1 180 637.039	+ 0.0019	0.0828	22.6196	− 0.6711	− 19.5473	1977.1401
35° 00′	391 284.623	208 715.377	1 125 169.185	+ 0.0017	0.0838	22.4810	− 0.6754	− 20.0453	2002.1639
30	390 725.790	209 274.210	1 069 696.891	+ 0.0016	0.0849	22.3406	− 0.6795	− 20.5465	2027.0351
36° 00′	390 160.033	209 839.967	1 014 220.132	+ 0.0014	0.0859	22.1986	− 0.6835	− 21.0507	2051.7518
30	389 587.393	210 412.607	958 738.878	+ 0.0013	0.0868	22.0548	− 0.6872	− 21.5577	2076.3120
37° 00′	389 007.912	210 992.088	903 253.106	+ 0.0012	0.0878	21.9095	− 0.6907	− 22.0673	2100.7140
30	388 421.634	211 578.366	847 762.790	+ 0.0010	0.0888	21.7624	− 0.6939	− 22.5796	2124.9558
38° 00′	387 828.601	212 171.399	792 267.910	+ 0.0009	0.0898	21.6138	− 0.6970	− 23.0944	2149.0357

West Zone to East Zone $E_0 = E'_0$ west $E'_0 = E'_0$ East k_2, k_6 positive, k_3 negative

East Zone to West Zone $E_0 = E'_0$ east $E'_0 = E'_0$ West k_2, k_6 negative, k_3 positive

ANNEXURE H

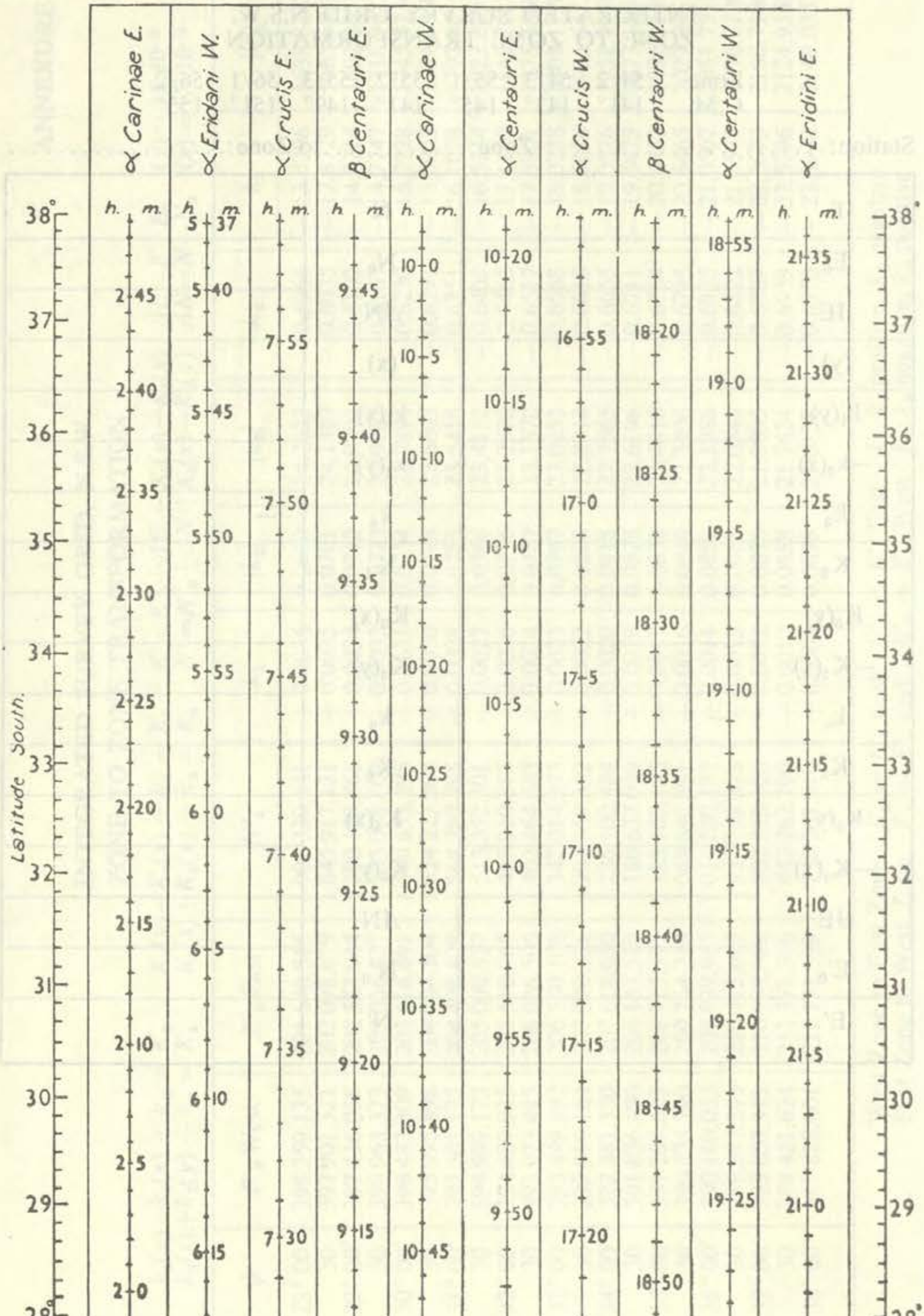
INTEGRATED SURVEY GRID N.S.W.
ZONE TO ZONE TRANSFORMATION

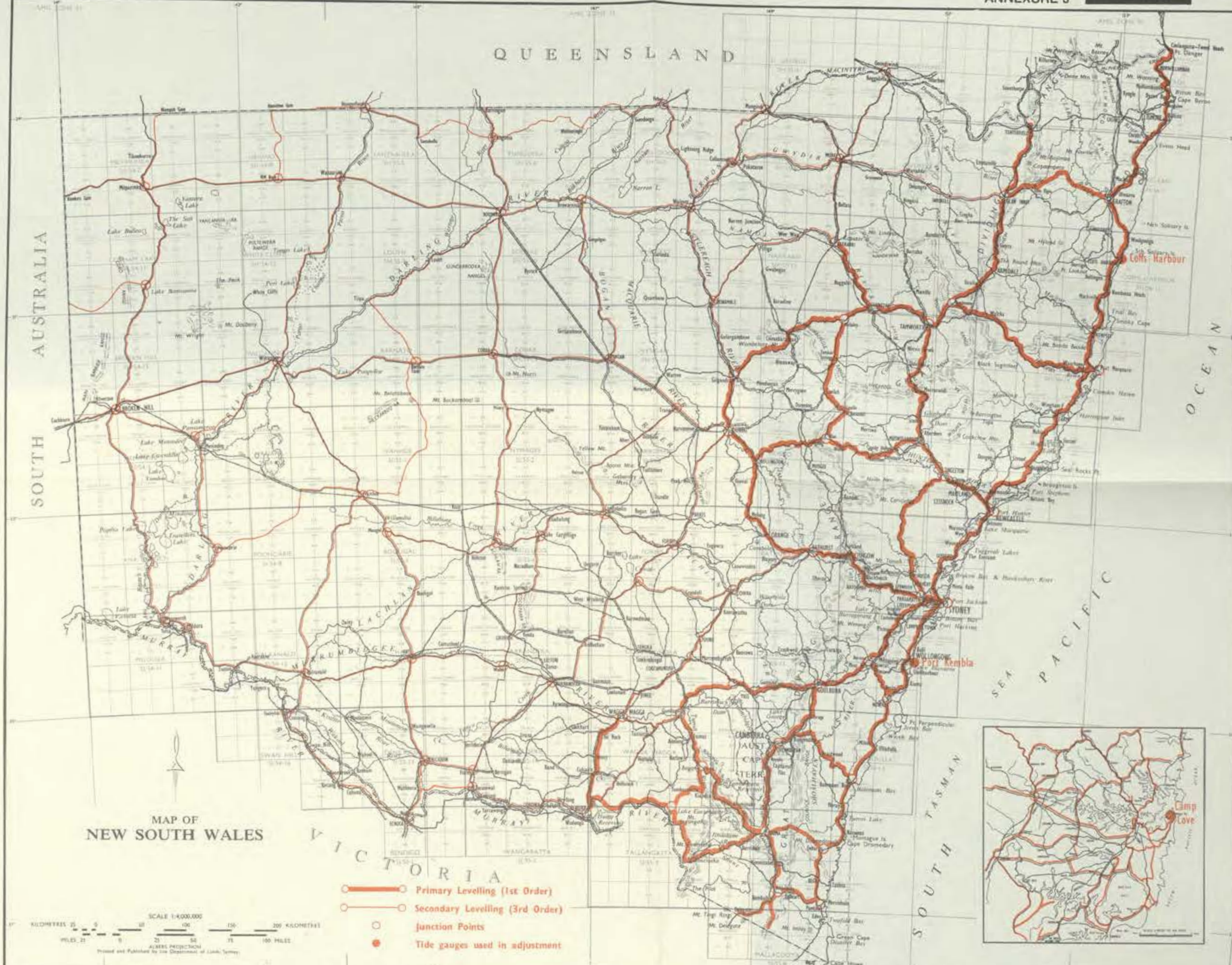
Zone	54/2	54/3	55/1	55/2	55/3	56/1	56/2
C.M.	141°	143°	145°	147°	149°	151°	153°

Station:........................Zone:............to Zone:..................

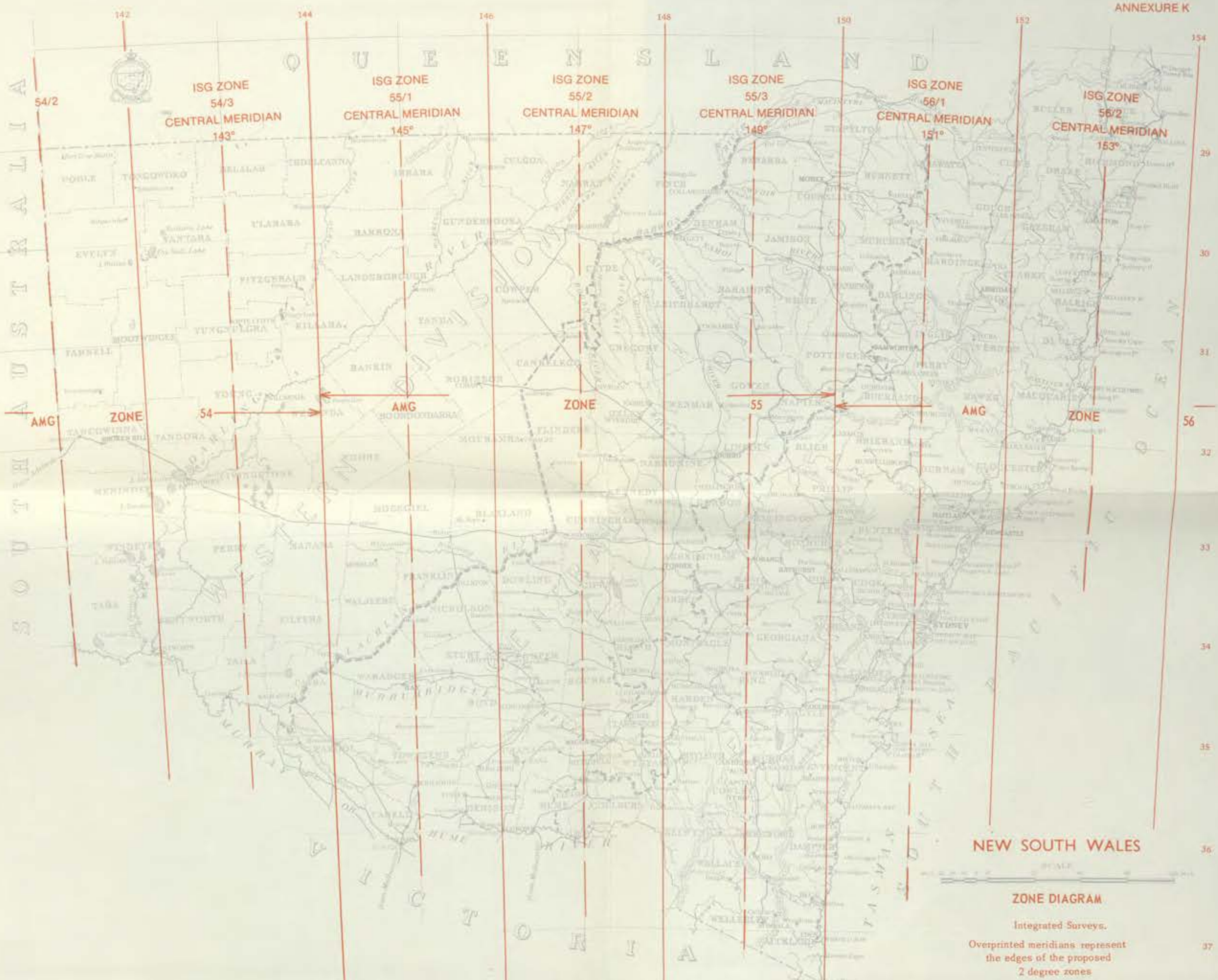
E		N	
E_0		N_0	
ΔE		ΔN	
(y)		(x)	
$k_1(y)$		$k_1(x)$	
$-k_2(x)$		$k_2(y)$	
k_3		k_4	—
K_3		K_4	
$K_3(y)$		$K_3(x)$	
$-K_4(x)$		$K_4(y)$	
k_5	—	k_6	
K_5		K_6	
$K_5(y)$		$K_5(x)$	
$-K_6(x)$		$K_6(y)$	
ΔE		ΔN	
E'_0		N'_0	
E′		N′	

LOCAL SIDERAL TIME AT ELONGATION




MAP OF
NEW SOUTH WALES
QUEENSLAND
SOUTH AUSTRALIA
VICTORIA
SOUTH TASMAN SEA
PACIFIC OCEAN
Coffs Harbour
Port Kembla
Camp Cove
SYDNEY
NEWCASTLE
WOLLONGONG
CANBERRA
BROKEN HILL
WAGGA WAGGA
TAMWORTH
ARMIDALE
Primary Levelling (1st Order)
Secondary Levelling (3rd Order)
Junction Points
Tide gauges used in adjustment
SCALE 1:4,000,000
KILOMETRES 25 0 50 100 150 200 KILOMETRES
MILES 25 0 25 50 75 100 MILES
ALBERS PROJECTION
Printed and Published by the Department of Lands, Sydney

ANNEXURE K



NEW SOUTH WALES

ZONE DIAGRAM

Integrated Surveys.

Overprinted meridians represent the edges of the proposed 2 degree zones