INSTRUCTIONS ON THE VERIFICATION OF ELECTRO-OPTICAL SHORT-RANGE DISTANCE METERS ON SUBSIDIARY STANDARDS OF LENGTH IN THE FORM OF EDM CALIBRATION BASELINES.

Proposed by Dr. J.M.Rüeger, April 1984.

SCHOOL OF SURVEYING

THE UNIVERSITY OF NEW SOUTH WALES

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Proposed by

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April, 1984.

1. - INTRODUCTION

These instructions contain the field and analysis procedures for the verification of electro-optical short-range distance meters on subsidiary standards of lengths in the form of EDM calibration baselines certified under Regulation 80, Weight and Measures (National Standards) Regulations 1966. They have been designed to produce instrument corrections in fulfillment of the requirements of Recommendation No.8 as made by the National Standard Commission's Working Party on the "Calibration of Electromagnetic Distance Measuring (E.D.M.) Equipment" on 1 February 1983:

"8. It is recommended that, in general, the minimum standard for uncertainty of calibration of an EDM instrument, assuming calibration against a monumented base, should be:

$$\pm (3 + 30 \times 10^{-3} \times L) \text{ mm}$$

at the 99% confidence level, where L is the interval length in metres".

(It should be noted that it is proposed to change the constant term in above equation to 5mm!).

The field and analysis procedures are the result of discussions held by the author with the verifying authorities of all States and Territories. The views of States and Territories have been taken into account, as far as a consensus was achieved. The suggested procedures are thought to constitute the minimal steps necessary to yield an instrument correction (usually comprising an additive constant, a scale correction and,possibly, cyclic error corrections) with a total uncertainty which is in agreement with the standard clause of Recommendation No.8 listed above. Sometimes, alternative procedures are listed which will yield instrument corrections of higher accuracy (lesser uncertainty).

These instructions also contain a list of items which should be stated on any type of certificates issued for a tested instrument. The stated items fully describe the instrument correction, its uncertainty of one part in one hundred, the conditions under which it has been measured and those, under which it is valid.

The author suggests that the States and Territories:

(1) publish the recommended field procedures of Section 2 in one form or another for issue to all surveyors in the state/territory. This would be in connection with informations on certified baselines, computing service, and repetition rate of calibrations. Such a publication should include standard field forms for the baseline measurement and the calibration of the thermometer (s) and barometer (s).

(2) establish a computer program along the lines of Section 3 for the processing of all baseline measurements in the State/Territory for the purpose of computing an instrument correction and its associated confidence intervalpursuant to Recommendation No.8.

(3) issue standardized test reports for all baseline measurements supplied, as recommended in Section 4. Should the measurements have been executed by the verifying authority itself, the same format can be used to issue a formal 'Regulation 80 Certificate'. The test reports should be programmed as part of the computer program listed above.

Because the test reports (andthe measurement and analysis procedures on which they are based) should be reciprocally acceptable, a uniform approach across Australia to the interpretation of Recommendation 8 is essential.

2. RECOMMENDED FIELD PROCEDURES

Below the minimal procedures are listed which have to be followed when verifying (calibrating) a short-range distance meter on a subsidiary standard of length (in the form of EDM calibration baseline, certified under Regulation 80, Weights and Measures (National Standard) Regulations, 1966) pursuant to Recommendation No.8, National Standard Commission's Working Party on the "Calibration of E.D.M. Equipment", dated 1 February 1983. Improved procedures for increased precision are listed as options. Occasionally, reasons for particular requirements are given. These recommended field procedures apply to short-range electro-optical distance meters, only.

2.1. - MEASUREMENTS ON BASELINES WITH PILLARS

Unless stated otherwise below, the measuring procedures specified by the respective manufacturer should be followed. However, should a more sophisticated measuring procedure be generally used with a particular distance meter, same sophisticated procedures should be followed on baselines if the calibration is to reflect these sophisticated procedures.

(1) - The observation sequence should be chosen in such a way that short lines are measured first and long lines later. (For example, if all 21 combinations on a 7 station baseline are to be measured: 6→7, 5→7, 5→6, 4→6, 4→5, 4→7, 4→8, 3→8, ..., 1→5, 1→4, 1→3, 1→2. (On Sprent-Zwart baselines, the reflector will move from the closest to the most distant station and back).

Reason: By the time long distances are measured, the frequency has stabilised because of levelling out of acclimatisation and warm-up processes.

(2) - The EDM instrument station must be shaded by an umbrella.

Reason: Temperature and barometers must be measured in the shade. The scale of an EDM instrument is temperature dependent and must be referenced against ambient air temperature (as measured in the shade). Furthermore, more and more manufacturers specify now the use of umbrellas on hot days and for more precise work. (An umbrella costs about $50).

Option: Shade both, instrument AND reflector station with an umbrella, as soon as temperatures are measured at both ends.

Note: The verifying authorities may provide permanent fixtures at instrument stations for easy use of survey umbrellas or, alternatively, install permanent sunroofs.

(3) - Temperature and pressure is measured (in the shade) at the instrument station at the beginning of baseline measurements, upon completion of these and at half-hourly intervals between the two times and at other times when a change in weather is experienced.

Reason: The scale of EDM is dependent on temperature and pressure through the velocity of light at the rate of 1 ppm per degree Celsius and per 3 millibars, respectively.

Note 1: The author decided against using pressures based on standard atmospheres. In Sydney, the difference between minimum and maximum pressure amounts to 57.5 mb. Using an average pressure would introduce a scale uncertainty (99% confidence level) of ± 10 ppm. This is excessive and would jeopardize all efforts to derive a scale versus temperature behaviour in the long term (from repeated baseline measurements). A small averoid barometer costs about $100. (Surely, all surveyors can afford such an outlay when buying a basic EDM outfit!)

Option: Temperatures and pressures are measured at both ends of lines (in the shade) and before and after the distance measurements on a line.

Note 2: In the case of steep baselines, the change of pressure between instrument and target station can be taken into account by the analysis program. Also for particular baselines, the verifying authorities may specify, at what stations the temperatures/pressures should be measured, based on the 30 minute intervals.

(4) - Partial Water vapour pressure or relative humidity is not measured. A baseline specific yearly average should be taken into account by the analysis.

Reason: The maximum effect of changes in humidity affects the first velocity correction by less than 2 ppm.

Note 1: The verifying authorities should evaluate average water vapour pressures for all baseline sites from information available at Regional Offices of the Bureau of Meteorology. For example, the partial Water vapour pressure (3pm) at Richmond, N.S.W., varies from 3.3.mb to 31.0 mb and has a mean of 12.8 mb. The maximum scale error caused by adopting a mean value is 0.73 ppm in this case. (This was derived from data given in: BUREAU OF METEOROLOGY, 1979. Climatic Survey, Sydney, Region 5, New South Wales, Australian Government Publishing Service, Canberra, Cat. No. 79 90989). See addresses in Appendix B.

Note 2: Alternatively, the same data may be evaluated from information given by the daily press under such headings as 'yesterdays weather',at least in major cities. A full year should be evaluated. The paper should state relative humidity and dry bulb temperature measured <u>at the same time</u> (and at same location). Such information may be found in 'THE SUN' (Melbourne), 'THE SYDNEY MORNING HERALD', 'THE CANBERRA TIMES', 'THE COURIER MAIL' (Brisbane), to give just a few examples. It is again stressed that humidity and dry bulb temperature must be given for same time, e.g. 9 a.m. or 3 p.m.

Note 3: The equation for the computation of partial water pressure from relative humidity is given in Rüeger,J.M. 1982. 'Introduction to Electronic Distance Measurement', as are tables for the saturation water pressure.

(5) - On the first EDM instrument station to be occupied, the EDM instrument is set up and shaded (without turning on) at least 15 minutes prior to measurement of the first line on the baseline.

Option: Set-up,turn-on and leave then for 15 minutes before taking the first measurement. The instrument is then kept operating all the time (even during transport) till the last line on the baseline has been measured.

Reason: Time should be given to the instrument to adapt (from transport) to ambient condition (acclimatisation). If this is done whilst operating, some warm-up effects are also overcome.

Important: EDM instruments equipped with oven controlled crystal oscillators must be warmed up as specified by their manufacturers.

(6) - The "ppm-knob" is set to the neutral position, usually 0ppm.

Reason: It is more accurate to apply the correction by computation, because the dial itself may not be linear and because the dial can be set with loss of accuracy only. (Step intervals can be as large as 30 ppm !).

(7) - On each line, four distance measurements are taken, with repointing after each measurement. Pointing is "optically" or "electronically" (maximum signal strength) as prescribed by the manufacturer of the instrument concerned.

Reason: Minimises and randomises pointing errors.

Option: Series of measurements are taken on each of the four pointings.

(8) - If possible, all distances are measured with one single prism. If necessary, longer distances may be measured to one triple prism. This (these) prism(s) should carry a permanent and unique identiciation label(s).

Note 1: It is suggested that the verifying authorities include a blank field on standardised field form for a sketch of relector assemblies used. In particular, horizontal axes should be shown. If the reflector tilts about an eccentric axis, the user should clearly state if use was made of this mechanism.

Note 2: Any other prisms generally used with the EDM instrument should be tested separately for differential reflector constants. This can be easily done, by measuring in turn to all prisms over about 100 m and repeating this 10 to 20 times (viz. A,B,C,D, A,B,C,D,...)

Note 3: The analysis program must be able to handle two heights of reflector per instrument, viz. one for the single prism and one for the triple prism.

(9) - The measurements should be executed with the attenuator or aperture setting as prescribed by the instrument's manufacturer for a particular distance range.

(10) - Between the first and the last measurement on the baseline, the EDM instrument should be kept in the open air and in the shade.

Reason: The temperature of the EDM instrument should be kept at ambient air temperature. See (2).

(11) - EDM instruments should be operated according to the manufacturer's instruction and/or according to the measuring procedures followed during "legal surveys". Should the manufacturer suggest a number of alternative procedures, the field notes and the verification certificate should clearly indicate which procedure was followed.

(12) - The EDM instrument should be turned on immediately prior to the four measurements of a line and it should be turned off after these four measurements.

Important: Instruments featuring over-controlled oscillators should not be turned off between the first and the last line measured on the base. See also (5).

Option: Best precision is achieved by running the EDM instrument continuously from the first line measured on a baseline to the last.

(13) - On pillared baselines, the height of the reflector, of any tilting axis on target/reflector assembly, of the EDM instrument and of the height of theodolite (if EDM equipment mounted on such) above the bottom plate of the tribrach used should be measured and booked to millimetres before and after the baseline measurements. This is best done in the office on a table, by measuring the heights from the table top with a pocket tape in four cardinal directions. The footscrews should be in mid-position during these measurements. Before levelling theodolite, EDM instrument and reflector on any station, the footscrews should be returned to the mid-position.

Reason: If levelling is started from mid-position of footscrews on all stations, the heights of the three equipments will be effectively the same on all stations.

(14) - Whenever spot bubles (circular levels) are used for levelling purposes, which cannot be rotated through 180° about the vertical axis of the respective equipment, they need to be checked and adjusted, if necessary, against a plate level of a theodolite before commencing the first measurement on a baseline and again after completion of all measurements on baseline.

Reason : Maladjusted spot bubbles cause errors in distance measurement by causing eccentricity errors. If the position of the spot bubble relative to the lines measured is always the same, a wrong additive constant will be determined. Otherwise, the random error component will be increased.

(15) - The temperatures should be measured at instrument height and in the shade with good quality mercury thermometers or with platinum resistance or thermistor thermometers designed for temperature measurements of gases. Temperatures should be read to one degree celsius. The error of the thermometer used, at the points of the scale used during the baseline measurements, shall not exceed ± 1.0°C. The thermometer(s) used shall carry a permanent and unique identification label.

Note 1: The accuracy of a thermometer should be ascertained at least once through calibration by a N.A.T.A. registered laboratory or by a comparison with a dry bulb thermometer employed by the Bureau of Meteorology inside the Stevenson's Screens at all weather stations. The thermometers used by the Bureau of Meteorology are in error by less than 0.15°C (above freezing point) (Refer to Australian Standard R13-1966).

Note 2: The addresses of N.A.T.A. labs may be obtained from National Association of Testing Authorities, 688 Pacific Highway, CHATSWOOD, N.S.W. 2067. The addresses of weather stations of the Bureau of Meteorology may be obtained from the Regional Directors. Addresses are given in Appendix B. Prior arrangements with the weather station keeper need to be made.

Option: The temperatures are measured to 0.1 to 0.2° C.

(16) - The pressures should be measured in the shade and with a horizontal barometer carrying a permanent and unique identification label. Aneroid barometers, should be graduated to at least 2 millibar and should be gently tapped prior to reading. Pressures should be read to at least 2 millibar.

The barometers should be calibrated against a mercury column barometer at least prior to baseline measurements. If a malfunction is detected on the baseline, a second calibration after the baseline measurements should be executed.

Mercury column barometers may be found at a large number of weather stations of the Bureau of Meteorology. Comparisons should be made against *'station level pressure'* and NOT against *'sea level pressure'!*

Note 1: If is usually safer for the barometers to be left on the ground rather than on the pillar. (It cannot fall down this way (but may be walked upon)). The systematic error caused in the scale of the EDM instrument by measuring pressures typically 1.5 m below the terminals amounts to 0.05ppm only.

Note 2: The addresses of Bureau of Meteorology weather stations may be obtained from its Regional Offices. (Refer to Appendix B). When making use of weather station facilities, prior arrangements should be made with the station keeper concerned. The mercury barometers used by the Bureau of Meteorology are in error by less than 0.4mb (Appendix B).

Note 3: The Regional Offices of the Bureau of Meteorology (in Capital cities) provide a calibration service for barometers (against a fee) as do some N.A.T.A. laboratories. The addresses of latter may be obtained from the National Association of Testing Authorities, 688 Pacific Highway, CHATSWOOD, N.S.W. 2067.

(17) - In the case of telescope or theodolite mounted EDM instruments, the axis of the EDM beam should be adjusted according to the manufacturers instructions. Under no circumstances should this adjustment be changed during the baseline test.

(18) - For theodolite (standard) mounted instruments the field form should clearly state, if the EDM instrument was attached in the 'face left' or 'face right' position of the theodolite (when EDM instrument and theodolite telescope are pointing to reflector).

Reason : The additive constant may differ between a FL mounting and a FR mounting.

(19) - The levelling of theodolite, EDM instrument and reflector is critical and should be done with utmost care. The field form should feature a field, which must be ticked upon completion of levelling.

Reason : Omission of levelling leads to errors in distance, which tend to increase random errors. Omission of levelling of EDM instrument in general, and at a terminal station of an all-combination-baseline in particular, leads to erroneous results. In the latter case, pillar "movements" will be suspected.

(20) - Baseline measurements should be carried out either fully during day-time or fully during night-time.

Note: If a EDM instrument is typically used at day-time, it should be calibrated at day-time. Instruments typically used in underground mines, should be calibrated at night.

Reason: The additive constant may differ between day-time and night-time operation.

2.2. - ADDITIONAL REQUIREMENTS FOR MEASUREMENTS ON BASELINES WITH GROUND MARKS.

In cases where unpillared baselines with groundmarks are used, the following additional rules are to be observed:

(21) Constrained centring is to be used on all stations, thus requiring one tripod per baseline station.

(22) Centring of tripods should be done by optical plumbing, preferably with an optical plummet which is built in to the upper plate (alidade) of a theodolite and which is, therefore, rotatable about the vertical axis.

Note: The stability of the tripods during the test is important and can be improved by setting the tripod's legs on pegs or on little concrete sockets and by shading all tripods throughout a test.

(23) When using the fixed optical plummets of tribrachs, tripods should be set up in such a way that the spot bubble of the tribrachs attached to them points to station 1 of the baseline. The tribrachs should be permanently numbered. Their spot bubbles and optical plummets should be checked and, where necessary, adjusted prior to the baseline measurement. After the baseline measurements, both spot bubbles and plummets should be checked again.

Reason: Should any change in bubble or optical plummet be found, the change can be measured and its effect on the baseline data established, as it is known where each tribrach was set up and how it was orientated.

Note: Special 'tribrach-tester' may be available at certain locations.

Option: The centring may be executed by plumbing using a theodolite at right angle to the baseline.

(24) The height of the reflector and of any tilting axis in the case of target/reflector assemblies should be measured at the time of any tripod occupation, viz. once per EDM line measured (to mm).

(25) The height of the EDM instrument (and the height of the theodolite) should be measured twice, namely when occupying a tripod and before leaving it (to mm).

Option: The verifying authority may place three concrete sockets each (at 120 degrees to each other, at same distance from the centre mark, at same elevation as centre mark) about all stations of a baseline. When using the same type of tripod on all stations, a good precentring is achieved. When using fully extended legs on all stations, same heights of instrument, reflector, .. will be achieved on all stations, thus reducing a likely source of error.

(26) Before dismantling the tribrachs and tripods, levelling and centring of the tribrachs should be checked and recorded. Any error in centring should be determined in terms of mm in the "along-baseline" component.

Option: Levelling and centring of tribrachs may be checked more frequently, namely before and after each occupation of a station.

(27) Care should be taken not to disturb tripods and tribrachs when shifting the reflector between stations.

(28) When occupying a new station with the EDM instrument, centring and levelling is to be checked and corrected, where necessary.

Reason: When measuring all combinations of a base, the EDM instrument will be the first equipment to occupy all but the last station (See (1)).

2.3. - MEASUREMENTS ON SEPARATE CYCLIC ERROR TEST LINES.

When determining short-periodic errors of EDM instruments (usually referred to as 'cyclic' errors) separate from the baseline measurements, some additional rules apply. In case that short periodic errors are derived from equally spaced reflector positions on a graduated steel tape (under tension, on a flat and frictionless surface), the following rules should be observed:

(29) The reference tape should be suspended on the support structure at least half an hour before commencement of measurement. At the same time, the support structure and the tape should be shaded.

Reason: If support and tape are truly in the shade and had time to adapt to ambient air temperature, the latter may be taken as tape temperature. Shading also overcomes the problem of partly shaded tapes. Shade temperatures are likely to be closer to the standard temperature of a tape than "sun" temperatures.

(30) The tape should be securely attached to the structure at the zero end and should be loaded by a free hanging weight at the other end, via a frictionless pulley.

(31) The spacing of the relevant graduations of the reference tape should be traceable to the National Standards.

Note: This can be achieved by getting a Reg. 80 certificate for the tape and all relevant graduations and the weight concerned. Alternatively, it may be measured with a Mekometer, as certified under Reg. 80, and become a subsidiary standard of length that way.

(32) Distances should be measured to twenty tape graduations equally spaced over the full unit length of the distance meter to be tested.

Definition: The 'unit length' of an EDM instrument is equivalent to one half of the wavelength of the modulation signal used for the 'fine measurement' of highest distance resolution. Typically, the fine modulation frequency is 15 MHZ, the modulation wavelength 20m and the unit length 10 m.

(33) The preferred measuring sequence is 1,3,5,7,...15,17,19,18,16, ... 6,4,2, with 1 and 20 being the marks closest to and furthest from the fixed zero end of the tape.

Reason: This gives best agreement between forward and backward run in case of a gradual temperature increase in the tape. It also eliminates a gradual change in the set up of the EDM instrument.

(34) Temperatures and pressures should be measured periodically at the instrument station and in the shade; for example, during measurements "1", "11", "20", "10" and "2".

(35) Four (4) observations should be made per reflector position with repointing in between.

Reason: This increases the precision and randomises pointing errors.

(36) Accurate levelling of the reflector is important, as is its centring.

(37) Furthermore, the following paragraphs apply : (2), (3), (5), (6), (8), (11), (12), (13) and (14).

(38) The EDM instrument should be set up in such a way that the EDM beam is truly colinear with all reflector positions on the tape.

Reason: No slope and offset corrections are thus required.

(39) The EDM instrument should be set up between 50m and 100m from the closest reflector position if short-periodic errors are to be determined at one range only.

Reason: On closer range, the non-periodic errors may overshadow the periodic errors. On longer range, the random errors become large when compared to the expected magnitude of short periodic errors.

(40) All distances of a short-periodic error test must be executed in one and the same attenuator/diaphragm position, as chosen from instrument handbook instructions.

Reason: Different attenuator/diaphragm settings can change the magnitude of short-periodic errors (and the additive constant).

2.4. CALIBRATION OF THERMOMETERS AND BAROMETERS

2.4.1. Calibration of an Aneroid Barometer

(41) When calibrating an aneroid barometer against a mercury column barometer (such as those kept at weather stations of the Bureau of Meteorology), the aneroid barometer should be set up at the elevation of the mercury container of the mercury column barometer and given about 10 minutes time to settle in the new environment. Make then two readings of the mercury barometer (column to 0.1 mb, temperature to 0.1^{o}C), four readings of the aneroid barometer (to 0.1 division) (with gentle tapping prior to each reading) and another two readings of the mercury barometer (pressure /temperature).

(42) The mean of the four mercury barometer readings may then be computed (including its standard deviation) and then be reduced to *'station level pressure'* using the appropriate table supplied with the barometer. Under no circumstances should readings be reduced to *'sea-level pressures'!!!*

After computing the mean of the four aneroid barometer readings and its standard deviation, the calibration constant may be computed, together with its standard deviation and its uncertainty at the 99% confidence level.

2.4.2.- Calibration of Thermometers

(43) When calibrating a thermometer against a 'dry bulb' thermometer as used by the Bureau of Meteorology inside the Stevenson's Screens at the weather stations, the surveyor's thermometer should be set-up in the screen as close to the dry bulb thermometer as possible. Close the screen door. Let the thermometer adapt to new ambient conditions for at least 10 minutes.

(44) After the acclimatisation period has elapsed, open the screen door and read the two thermometers within 30 seconds of opening the screen door, in the following sequence:

reference thermometer
surveyors thermometer
reference thermometer
...

and so forth until a total of 4 reference thermometer and 4 surveyors' thermometer readings have been obtained. All temperatures should be read to the nearest tenth of a degree after bringing the eye level with the surface of the mercury in the thermometer stem.

(45) From the four pairs of observations, four calibration constants for the surveyor's thermometer can be calculated, as well as their mean, its standard deviation and its uncertainty at the 99% confidence level.

3. RECOMMENDED ANALYSIS PROCEDURES

3.1. UNDERLYING ASSUMPTIONS

The suggested analysis procedures are based on a few assumptions which are outlined here.

(1) The EDM calibration measurements have been carried out on an EDM baseline certified under Regulation 80 of the National Standards Regulations by the Verifying Authority of the State or Territory. This implies that the baseline and, possibly, the attached cyclic error test facility are periodically measured with a distance meter, certified under Reg.80 of the National Standards Regulations.

(2) The measurements executed by the verifying authority, subsequently referred to as reference measurements, have been executed and processed according to the "Approved Method of Verification of a Base using the Mekometer", as foreshadowed in Recommendation No.4 of the working party "Calibration of EDM Equipment" of the National Standards Commission (Meeting, 1.2.1983).

(3) The reference data are available in two forms (pillared baselines are assumed):

(3.1) Slope distances as measured and as corrected for the first velocity correction and the frequency correction. The first velocity correction should be based on a velocity of light of 299 792 458 m/s and on the current refractive index formulae recommended by the International Association of Geodesy, most recently in 1963. The carrier wavelength employed should be stated. The temperatures and pressures (including partial water vapor pressures) used in the first velocity corrections should include their respective calibration values. Accurate values for height of EDM instrument and height of reflector should be given. The standard deviation of each mean distance should be stated, together with the degree of freedom. The levelling precision of EDM instrument and reflector should be evaluated and given.

(3.2) Adjusted horizontal baseline distances with associated variance-covariance matrix. These must refer to a horizontal reference surface clearly specified.

Note: In special circumstances, the adjusted baseline distances may refer to a sloping line, in which case the elevation of both stations which define the slope must be clearly stated.

(3.3) In both cases, the following additional details must be available:

(i) Elevations of all stations above a reference level surface.

Note: These elevations must be given to such accuracy so as not to downgrade any distance by the "slope correction".

(ii) Offsets of all stations from a vertical plane through the first and last stations.

Note: These offsets must be given to such an accuracy that the offset corrections do not downgrade the reduced distances.

(iii) Dates of distance, elevations and offset measurements, respectively.

(iv) Centring accuracy of pillar centring device or, alternatively, centring accuracy within tribrach and centring accuracy between tripod mounted tribrach and ground mark.

(4) The marks of the baseline do not move along the baseline, across the baseline or vertically between repeated measurements of the reference data sets (distances, offsets, elevations).

3.2. PRE-PROCESSING OF DISTANCE MEASUREMENTS

(1) If the sum of EDM instrument additive constant and of reflector (additive) constant, as previously known, exceeds a few centimetres, it should be added to all measurements (= a priori additive constant).

(2) For each line measured the mean (of four) distance measurements is computed as well as the standard deviation of a single distance measurement (S_D).

(3) The standard deviations (of single distance observations) from all lines are plotted versus distance. A simple linear regression leads to the "a priori" standard deviation of distances:

$$S_D = \pm (A \text{ mm} + B \text{ ppm}) \qquad (1)$$

Reason: As the manufacturer's specifications are totally unsuitable for the estimation of the precision of distance measurements on baselines, this is done on the basis of the actual measurements. The fact that the mean of four observations is introduced into the subsequent adjustment is ignored, as the four observations are (physically) highly correlated.

(4) For each line, the corresponding temperature is interpolated (according to time) from the temperatures measured, at, typically, 30 minutes intervals and corrected for the calibration value.

Option: For each line, the temperatures measured at the instrument end and at the reflector end, respectively, are meaned. Then, the line mean temperature is computed.

(5) An estimate of the standard deviation $S_{\bar{T}}$ of these interpolated temperatures should be evaluated and should include: the measurement precision, the uncertainty caused by measuring at one end only (including height effect on sloping lines) and the error of interpolation over typically 30 minutes.

Note 1: The verifying authorities should establish typical values for individual baselines, preferably at four distinct atmospheric conditions: (1) sunny, no wind; (2) sunny with wind; (3) overcast, no wind; (4) overcast, windy.

Example: Three data sets of the UNSW EDM Research Baseline at Regent Park, Sydney, have been analysed. This baseline is 980m long and features a total height difference of 12m. Temperatures were measured at both ends of lines to 0.1°C; the means were then compared to interpolated '30 minute temperatures'. On a sunny to overcast day, with moderate to no wind, an $S_{\bar{T}}$ of ± 0.5° C resulted. On a sunny day with gusty to no wind: $S_{\bar{T}}$ = ± 0.7°C; fully overcast day with no wind: $S_{\bar{T}}$ = ± 0.3° C. (These values include a measurement precision of ± 0.1°C).

Option: The standard deviation of a line mean temperature is computed from actual measurements, provided that temperatures have been measured for all lines and at both terminals.

(6) After applying the calibration constant to all barometer readings, the pressure readings for all lines may be obtained by interpolation according to time. On sloping baselines, the change of pressure with height may have to be taken into account.

Reason: Atmospheric pressure changes by -0.120mb/m at sea level and by -0.108mb/m at an elevation of 1000m. In terms of EDM scale this amounts to 0.034ppm/m.

Note: Interpolation and correction for elevation may be carried out by the computer program, provided that times and the elevations of barometer positions are entered.

Option: For each line, the pressures measured at the endpoints are corrected for their respective calibration values. Then, the line mean pressure is computed.

(7) An estimate of the standard deviation $S_{\bar{p}}$ of these interpolated pressures should be evaluated and should include the precision of a pressure reading and the error of interpolation over typically 30 minutes. The uncertainty caused by measuring only at one end should also be included, if the pressure change with height is not taken into account by computation.

Example: The random error caused by interpolation between 30 minute pressure readings and omission of the change of pressure with height has been computed as $S_{\bar{p}}$ = ± 0.3mb for a Regents Park data set. (Total height difference: 12m). The standard deviation of a single pressure reading was evaluated as ± 0.1mb (THOMMEN barometers type 2A2.511.02) and is included in above value.

Option: The standard deviation of a line mean pressure is derived from actual pressure readings at both terminals of all lines. (Using above example: $S_{\bar{p}}$ = ± 0.06mb).

(8) A first velocity correction K' of the form

$$K' = \left(C - \frac{D \times p}{(273.15 + t)} + \frac{11.20 \times e}{(273.15 + t)}\right) 10^{-6} d \qquad (2)$$

where t = line mean temperature (in ^{o}C) (refer to (4))
p = line mean pressure (in mb) (refer to (6))
e = partial water vapour pressure (in mb)
C = instrument parameter
D = instrument parameter
d = measured distance

is added to all mean distances of paragraph (2). An early average value of the partial water vapour pressure e is used as determined by the verifying authority for a particular baseline site (Refer to (4) in Section 2.1.).

The parameters C and D are specific for instrument types and may be derived on the basis of the nominal unit length, the nominal modulation frequency, the nominal carrier wavelength and the appropriate refraction index formula and the velocity of light as recommended by the International Association of Geodesy. Alternatively, C and D may be taken from the instrument handbook.

Note 1: Depending on the parameters used in the first velocity equation (2), different scale factors will be found. It is therefore imperative to list the equation used in full in any calibration report.

Option: If humidity readings were taken during the baseline measurements, a mean partial water vapour pressure ' $\bar{e}$ ' over all lines may be used in Eq.(2).(Although relative humidity changes during a day (with temperature), the absolute water content does not, at least not significantly. It is therefore appropriate to use the same average value $\bar{e}$ for all lines).

Note 2: The first velocity correction according to Eq.(2) may be applied by the analysis program, as soon as temperatures and pressures are provided as input data.

(9) Where necessary, lines are corrected for any eccentricities their respective terminals may have from the straight line between the first and last station.

(10) Where necessary, an additional correction has to be applied for EDM instrument which are attached to theodolites. However, no correction is required if:

(a) the EDM instrument is mounted on the standards of the theodolite and features its own trunnion axis provided that the tilting axis of the reflector is vertically above the mark. H_{EDM} and H_{REF} are measured according to the definition in Eq.(4).

(b) the EDM instrument is mounted on the telescope of the theodolite at an offset E and the reflector is mounted at an offset E above a target, with the reflector tilting about an axis which goes through the centre of the target, provided that the reflector/ target is always tilted to point at the theodelite, that the theodolite height is entered as H_{EDM} and the target's tilting axis as H_{REF} in Eq.(4) below.

For a telescope mounted EDM instrument measuring to a reflector with a tilting axis which intersects the plumbline through the mark, the additional correction is:

$$+\frac{E^2}{2\,d} \qquad (3)$$

where E is the vertical offset of the EDM instrument form the telescope axis and d is the measured distance, provided that the height of the theodolite is entered as H_{EDM} in Eq. (4) below.

Note: Refer to Sections 4.133 and 4.134 in RÜEGER(1982) for reference. The correction after Eq. (3) yields typically less than 1mm for distances in excess of 10m.

(11) All lines are reduced on the same horizontal reference elevation E_R:

$$HD_{i,j} = \left(D - \frac{\Delta H^2}{2D} - \frac{\Delta H^4}{8D^3} - \frac{\Delta H^6}{16D^5} + \frac{\bar{H}\Delta H^2}{2DR} + \frac{\bar{H}\Delta H^4}{8D^3R} + \frac{\bar{H}\Delta H^6}{16D^5R} - \frac{\bar{H}D}{R}\right)\left(1.0 + \frac{E_R}{R}\right) \qquad \ldots(4)$$

where D = mean distance of lines i to j as corrected (see paragraphs (2), (8), (9)).

$\bar{H} = 0.5\ (H_i + H_{EDM} + H_j + H_{REF})$

$\Delta H = (H_i + H_{EDM}) - (H_j + H_{REF})$

H_i = elevation of station "i" (instrument)(in m)

H_j = elevation of reflector station "j" (in m)

H_{EDM} = height of EDM instrument(in m) (see also (10) above)

H_{REF} = height of reflector (in m) (see also (10) above)

R = radius of the earth (in m)

HD_{ij} = horizontal distance from i to j at elevation E_R

Note 1: For reference, see RÜEGER (1982), Eqs. 3.22c and 3.33c. All horizontal distances HD are now colinear as far as the horizontal component is concerned and at exactly the same elevation (E_R).

Note 2: The analysis program should provide an input facility for two H_{REF}, namley a first one for a single reflector and a second one for a triple reflector.

(12) The a priori estimate of the standard deviation of the horizontal distances should appropriately model all random errors affecting the baseline measurements.(Any systematic uncertainties affecting all measurements in a similar way(such as calibration constants of the thermometers and barometers) have no effect on the least-squares adjustment and will be accounted for later!).

$$S_{HD} = \pm (\bar{A} \text{ mm} + \bar{B} \text{ ppm}) \quad (5)$$

where $$\bar{A}^2 = A^2 + 2S_c^2 + S^2_{L_{EDM}} + S^2_{L_{REF}} + 2S^2_{GM} \quad (6)$$

$$\bar{B}^2 = B^2 + (1.0\ S_{\bar{T}})^2 + (0.3\ S_{\bar{p}})^2 \quad (7)$$

A,B = see paragraph (3) and Eq. (1)

$S_{\bar{T}}$ = see paragraph (5) (in ^{o}C)

$S_{\bar{p}}$ = see paragraph (7) (in millibar)

S_c = standard deviation of centring EDM instrument or reflector on pillar or in forced centring device. Latter refers to tripod measurements (in millimetre)

$S_{L_{EDM}}$ = standard deviation in distance due to random errors in levelling of EDM instrument (in mm)

$S_{L_{REF}}$ = standard deviation in distance due to random errors in levelling of reflector (in mm)

S_{GM} = standard deviation of centring of tripod mounted tribrach over ground mark including tripod instability over time of baseline measurement. This applies only to baselines with ground marks.

A value for S_c may be stated for a baseline or may be established by a little experiment. Eccentricities caused by random levelling errors, and thus S_L, may also be found by experiment.

Note 1: The latter may be executed by setting up two tripods, in the shade, about 5m apart. A theodolite is set up on one, and a reflector on the other. Orientate reflector to theodolite. Re-level reflector and measure direction to apex of prism ten times in turn. The standard deviation of a direction times distance gives a suitable value for S_L.

Note 2: If a triple prism is used, which features a reflector height different from the single prism, then, strictly speaking, an S_L value for each would leave to be evaluated. It is suggested to use a pooled value in Eq.(6) above.

3.3 - PHILOSOPHY OF ANALYSIS

The aim of the least-square analysis is to produce the calibration parameters and their associated accuracies and uncertainties. The selection of a suitable set of calibration corrections must be based on the knowledge of instrumental errors. The number and type of corrections to be solved for will depend on instrument types and on individual instruments.

An instrument correction of the following mathematical form will model the systematic errors of most distance meters:

$$\begin{aligned} I.C. = {} & a_0 + a_1 D + a_2 D^2 + a_3 D^3 + a_4 D^4 + a_5 D^5 \\ & + b_{11} \sin B_1 + b_{12} \cos B_1 + b_{21} \sin B_2 + b_{22} \cos B_2 \\ & + b_{31} \sin B_3 + b_{32} \cos B_3 + b_{41} \sin B_4 + b_{42} \cos B_2 + \end{aligned} \quad \ldots (8)$$

where a = coefficients of a polynomial expression

b = amplitudes of periodic terms (cyclic errors)

B = argument of periodic terms (cyclic errors)

D = distance

Short periodic errors have arguments of:

$$B_1 = \frac{D}{U} 2\pi, \quad B_2 = \frac{2D}{U} 2\pi, \quad B_3 = \frac{3D}{U} 2\pi, \ldots \quad (8a)$$

where U stands for the unit length of the distance meters. (The'unit length' of an EDM instrument is equivalent to one half of the wavelength of modulation signal used for the 'fine measurement'. See also (32) in Section 2.3.). Pulse distance meters do not use modulated signals and are therefore unlikely to be affected by short periodic errors.

Sometimes, periodic errors of wavelengths in excess of U need to be modelled, namely long periodic errors. Usually, only the first two terms of the polynomial expression need to be considered, namely the additive constant "a_0" and the scale correction "a_1".

Recommendation No. 8 of the Working Party of the National Standards Commission on the"Calibration of E.D.M. Equipment" specifies that the 99% confidence level of the instrument correction I.C. (see Eq. (8)) shall not exceed.

$$\pm(3\text{mm} + 30\text{ppm}) \quad (9)$$

for any distance D in the range of distances tested. In other words, the probability P, that the I.C. lies within the confidence limits, can be expressed by:

$$P\,(I.C. - (3\text{mm} + 30\text{ppm}) < \overline{I.C.} < I.C. + (3\text{mm} + 30\text{ppm})) = 0.99 \quad \ldots (10)$$

where the sample instrument correction is denoted by I.C., the population parameter ('true value' of the instrument correction) by $\overline{I.C.}$

Indirectly, the specifications of Eqs. (9) and (10) will have a bearing on the choice of the number and type of terms in Eq. (8). The more terms of Eq. (8) are solved for, the smaller the residuals will become and the smaller the degree of freedom (number of observations minus number of unknowns). The 'a posteriori' variance factor, which is affected by the residuals as well as the degree of freedom may or may not become smaller with an increase of the terms solved for.

As the procedure is iterative in nature, the author suggests to obtain a first solution with the following set of calibration parameters:

$$\text{I.C. (1)} = a_0 + a_1 D + b_{11} \sin\left(\frac{D}{U} 2\pi\right) + b_{12} \cos\left(\frac{D}{U} 2\pi\right)$$

$$+ b_{21} \sin\left(\frac{2D}{U} 2\pi\right) + b_{22} \cos\left(\frac{2D}{U} 2\pi\right) \qquad (11)$$

This "first" instrument correction incorporates an additive constant, a scale correction and the first and second order short periodic error. If it is found that the I.C. (1) so determined fulfils Eqs. (9) and (10), the magnitude of the amplitudes b_{11}, b_{12}, b_{21}, b_{22} should be tested statistically against zero at the 95% confidence level. A second adjustment should follow, with the I.C. (1) model reduced by the number of insignificant periodic terms to I.C. (2). If the second instrument correcting I.C. (2) also fulfils Eqs. (9) and (10), this instrument correction should be certified.

Note 1: It is the author's view that a_0 and a_1 should always be solved for, even if they are statistically insignificant. As soon as a_0 is added to 'a priori' additive constants or built-in constants, they are always significant. The scale correction is dependent on temperature. To allow comparisons between different calibrations of the same instrument, likely executed at different temperatures, the computation of a_1 is required at all times.

Note 2: Although the statistical analysis for National Standard purposes is based on the 99% confidence level, it is suggested to test periodic errors at the 95% confidence level. The latter confidence level is more commonly used in surveying. If a periodic error is not significant at the 95% level, it will not be significa at the 99% level either.

If an instrument correction I.C. (1) after Eq. (11), determined for a particular instrument, does not fulfil the specifications of Eqs. (9) and (10), one can proceed in different ways. The most obvious approach is to use all parameters of Eq. (8) to model the instrument correction. This is only possible if a large number of redundant measurements is available. This, in turn, depends on the baseline design. The selection between a polynomial expression of 5th degree or long-periodic errors can be made on the basis of a plot of residuals versus distance. A plot of residuals versus unit length may indicate if 3rd and 4th order short-periodic errors should be solved for.

Alternatively approaches are to remeasure the instrument on a more sophisticated baseline, or to specifiy the instrument correction at an uncertainty worse than that listed in Eq. (9).

3.4. LEAST-SQUARES ANALYSIS

3.4.1. Observation Equations for Test Instrument Data

Each horizontal distance HD after paragraph (11) of Section 3.2. yields an observation equation of the following form:

$$HD_{ij} + v_{ij} + I.C. = X_j - X_i \qquad (12)$$

where X_k = adjusted coordinates of baseline station k, taking X_1 = 0.000m

v_{ij} = residual of horizontal distance HD_{ij}

Using the linearisation

$$X_k = x_k + X_k^o \qquad (13)$$

where X_k^o = approximate coord. of station "k" and again with $X_1 = X_1^o = x_1$ = 0.000, the following observation equations result:

$$v_{ij} = x_j - x_i\, a_0 - \left(\frac{HD_{ij}}{1000}\right) a_1 - \left(\frac{HD_{ij}}{1000}\right)^2 a_2^2 - \left(\frac{HD_{ij}}{1000}\right)^3 a_3^3$$
$$-\left(\frac{HD_{ij}}{1000}\right)^4 a_4^4 - \left(\frac{HD_{ij}}{1000}\right)^5 a_5^5 - \ldots$$
$$- (\sin B_1)\, b_{11} - (\cos B_1)\, b_{12} - (\sin B_2)\, b_{21} - (\cos B_2)\, b_{22} -$$
$$-(HD_{ij} - (X_j^o - X_i^o))\; 0.001 \qquad (14)$$

where all terms are in units of millimetres and HD_{ij}, X_j^o, X_i^o in metres. Unknown parameters are x_k, a_0 to a_5 and b_{11} to b_{22}. All but a_1 to a_5 are obtained in units of mm. The scale factor a_1 is obtained in ppm, for example.

Caution should be exercised when computing the argument B_n of the periodic terms:

$$B_n = 6.283185\left(\frac{D_{ij}\;(\text{MOD } L_n)}{L_n}\right) \qquad (15a)$$

where B_n is the argument of the n-th periodic term (in radian) and L_n the respective wavelength in metre. It should be clearly noted that the slope distance D_{ij} is employed in the above equation and not the horizontal distance HD_{ij}. For horizontal baselines, the difference between D_{ij} and HD_{ij} is trivial and can be ignored. For sloping baselines and long lines D_{ij}, the difference is far from trivial. The following equation is valid for horizontal baselines only:

$$B_n = 6.283185\left(\frac{HD_{ij}\;(\text{MOD } L_n)}{L_n}\right) \qquad (15b)$$

It has been mentioned before that, in a first step, a_2 to a_5 are not solved for and that L_1 and L_2 are taken as U and 0.5U respectively. (U = unit length of distance meter). The observation equations for the measurements of the distance meter to be calibrated are therefore fully described.

3.4.2. Observation Equation for Reference Distances

The question now arises how to consider the reference data as per paragraph 3.1, Section 3.1, or as per paragraph 3.2, Section 3.1. If the first reference data set is to be used, it is a simple matter to set up observation equations for the measurements with the "prescribed EDM equipment", presently the Kern Mekometer ME 3000:

$$v^*_{ij} = x_j - x_i - a^*_0 - 0.001\ (HD^*_{ij} - (X^o_j - X^o_i)\ ) \qquad (16)$$

where all parameters with an asterisk refer to reference measurements. In a form of self-calibration, the additive constant of the "prescribed EDM equipment" is solved for. This is possible as long as all or most combinations of lines on the baseline are measured with this "prescribed EDM equipment". With the Mekometer, the scale factor a^*_1 is determined by frequency measurements. All other parameters (a^*_2 to a^*_5, b^*_{11} to b^*_{22}) are believed to be zero for this type of distance meter. (If not, they need to be calibrated elsewhere and the necessary correction applied to the measurements before entering the analysis program). The use of the reference data in form of actually measured slope distances (as corrected according to paragraph 3.1 of Section 3.1) has proved very convenient in practice, particularly as far as programming is concerned.

One alternative method is to use adjusted horizontal distances as per paragraph 3.2 of Section 3.1. The observation equations for the coordinates yield in this case:

$$X^*_k + v^*_k = x_k + X^o_k$$

$$v^*_k = x_k - (X^*_k - X^o_k) \qquad (17)$$

where $X^*_1 = X_1 = X^o_1 = x_1 = 0.000$

and where the (adjusted) reference coordinate of station "k" and its residual are denoted by $X_k{}^*$ and v^*_k, respectively. One of the disadvantages of this approach is that a complete variance-covariance matrix of the vector of reference coordinates must be stored.

3.4.3. Observation Equation for Cyclic Error Testline Data

Additional observation equations for the EDM instrument to be tested are required whenever the short-periodic errors are determined on separate testlines against a calibrated tape, as outlined in Section 2.3.

$$D_m^+ + V_m^+ + b_{11}\ (\sin B_1) + b_{12}(\cos B_1) + b_{21}\ (\sin B_2) + b_{22}\ (\cos B_2)$$

$$= X_{CE} + (T_m - T_1) \qquad (18)$$

where D_m^+ = distance measured to reflector on "m" -th tape mark, including first velocity correction

V_m^+ = residual of D_m

X_{CE}= adjusted distance between EDM instrument and first reflector position over tape mark T_1

T_m-T_1= true length of tape between tape graduations $T_1$and T_m

All other parameters have been previously defined. Linearising with:

$$X_{CE} = X_{CE}^o + x_{CE} \qquad (19)$$

the final observation equation for distance measurements against a calibrated tape for the purpose of "cyclic" error determination yields:

$$V_m^+ = + x_{CE} - (\sin B_1)\ b_{11} - (\cos B_1)\ b_{12} - (\sin B_2\ )\ b_{21} - (\cos B_2)\ b_{22}$$

$$- 0.001\ (D_m^+ - (X_{CE}^o + T_m - T_1)) \qquad (20)$$

where the parameters D_m, X_{CE}^o, T_m, $T_1$are given in metres and all others in millimetres, where a "+" refers to cyclic error test observations, and where:

$$B_1 = 6.28319\ \left(\frac{D_m\,(MOD\ U)}{U}\right) \qquad B_3 = \ ....$$

$$B_2 = 6.28319\ \left(\frac{D_m\,(MOD\ (U/2))}{U/2}\right) \qquad B_4 = \ .... \qquad (21)$$

It is important to realise that the parameters b_{11} to b_{22} of Eq.(20) are exactly the same as those of Eq. (14).

3.4.4. 'A Priori' Weighting of Observations

The least-squares adjustment requires a good a priori estimate for the weight coefficient matrix P of the observations. No correlation between observations is taken into account. (The analysis program may however feature such an option for future use). The 'a priori' standard deviation for the baseline measurements are taken from Eq.(5) of Section 3.2 (paragraph 12). A similar estimate must be available for the reference data described in paragraph 3.1 of Section 3.1. An "a priori" estimate for the cyclic error testline measurements D_m^+ may be derived by an approach similar to paragraph 12 of Section 3.2. All observations D_m^+ are necessarily of equal precision.

The 'a priori'estimates of the standard deviations, of the two or three groups of observations must be checked, based on 'a posteriori' estimates. The 'a posteriori' variance factors of all groups must be the same and equal to one:

$$\frac{\Sigma vpv}{n_1} \left(\frac{n}{n-u}\right) = 1.0 \qquad (22a)$$

$$\frac{\Sigma v^* p^* v^*}{n_2} \left(\frac{n}{n-u}\right) = 1.0 \qquad (22b)$$

$$\frac{\Sigma v^+ p^+ v^+}{n_3} \left(\frac{n}{n-u}\right) = 1.0 \qquad (22c)$$

$$n = n_1 + n_2 + n_3 \qquad (22d)$$

where v, p refer to the n_1 measurements on the baseline with the distance meter to be tested, v^*, p^* to the n_2 reference measurements and v^+, p^+ to the n_3 measurements against a graduated tape for short periodic error determination. Weights, residuals and number of observation in a group are denoted by p, v and n, respectively. The total number of unknown parameters is denoted by u and depends on the number of stations on the baseline and the number of terms in the instrument correction I.C., amongst other things. Should any of the equations (22a) to (22c) not hold, then the a priori standard deviation should be adjusted accordingly and a second adjustment be executed.

3.4.5. Adjustment Procedure

Standard least-squares procedures yield the solution vector X:

$$X = (A^T PA)^{-1} A^T P\ell \qquad (23)$$

where A = matrix of coefficients of unknown parameters

P = (diagonal) weight matrix of observations

ℓ = vector of constant terms

X = vector of unknown parameters, incl. a_0 to a_5, b_{11} to b_{22}, x_2 to x_k, a_0^*, X_{CE} (where "k" = number of baseline stations),

the elements of the diagonal of the weight matrix P:

$$p = \frac{1.0}{S_{HD}^2} \qquad (24a)$$

(where S_{HD} has been defined in Eq. (5), paragraph (12), Section 3.2),

the vector v of residuals (in millimetre)

$$v = AX - \ell \qquad (24b)$$

the a posteriori variance factor:

$$\bar{\sigma}_o^2 = \frac{v^T Pv}{n-u} \qquad (25)$$

where n = total number of observations

u = total number of unknowns,

the cofactor matrix of the unknown parameters

$$Q_{xx} = (A^T PA)^{-1} \tag{26}$$

from which the variance-covariance matrix of X is obtained by multiplication with $\bar{\sigma}_o^2$. If Eqs. (22a) to (22c) hold, then the overall a posteriori variance factor $\bar{\sigma}_o^2$ (Eq. (25)) will also become one exactly.

The statistical significance of any of the unknown parameters can be checked by testing the null hypothesis $H_o: X_i = 0$. The null hypothesis is accepted if the test statistic T:

$$T = \frac{|X_i - 0|}{\bar{\sigma}_o \sqrt{q_{xx}}} < t_{(95\%,\ n-u)} \tag{27}$$

where t stands for t-distribution and q_{xx} for the diagonal element of the cofactor matrix of the unknown parameters which corresponds to the X_i to be tested.

Note: The reasons for the testing at the 95% confidence level (rather than 99%) have been given in Section 3.3.

3.5. STATISTICAL ANALYSIS

The derivation of the uncertainty of the instrument correction I.C. will be demonstrated on the basis of Eq. (11) of Section 3.3. The instrument correction is already expressed as a function of unknown parameters:

$$I.C.\ (1) = a_o + D\ a_1 + b_{11} \sin B_1 + b_{12} \cos B_1 + b_{21} \sin B_2 + b_{22} \cos B_2$$

For the error analysis, it is possible to restrict the derivation of the uncertainty to distances D being multiples of the unit length U. The coefficients of the sine terms thus become zero and those of the cosine terms one exactly.

$$F = a_0 + 0.001\ D\ a_1 + b_{12} + b_{22} \tag{28}$$

Note: a_0, b_{12} in mm; D in m; a_1 in ppm.

The vector f of partial derivatives yields:

$$f^T = (1.0,\ 0.001\ D,\ 1.0,\ 1.0) \tag{29}$$

The variance of the instrument correction I.C. for any distance D (being a multiple of the unit length U) follows as :

$$Q_{FF} = f^T\ Q_{xx}^f\ f \tag{30}$$

where Q_{xx}^f is a sub-matrix of the variance-covariance matrix of the unknown parameters comprising all variances of and covariances between the parameters a_0, a_1, b_{12} and b_{22}. The standard deviation of an I.C. (for a particular distance D) yields:

$$\sigma_{IC} = \sqrt{Q_{FF}} \tag{31}$$

at the 66% confidence level. The 99% confidence level is obtained by multiplication with $t_{99\%,n-u}$.

$$P\ (I.C. - t_{99\%,n-u}\ Q_{FF}^{0.5} < \overline{I.C.} < I.C. + t_{99\%,n-u}\ Q_{FF}^{0.5}) = 0.99 \quad \ldots(32)$$

where $\overline{I.C.}$ stands for the population parameter ('true' value). It is recommended to compute the confidence interval according to Eq. (32) for the shortest, the longest and for the mean (of all) distances as measured with the EDM instrumented to be tested and to list these in a table. As a service to the customers, the confidence interval may also be computed for distances equivalent to the double, triple and quadruple of the longest distance measured on the base. When listing these, they should be set in brackets, as they are based on extrapolation.

Note: Surveyors will want to know the uncertainties of the instrument correction for distances exceeding the maximum baseline distance. The instrument correction and its associated uncertainty can naturally be computed rigorously for any distance. Outside the range of baseline distances this is however based on the assumption, that the calibration parameters determined on the baseline also hold for distances below or above the range of distances tested. This may not be a valid assumption in all cases. (It has been suggested to double the uncertainties of the I.C. at distances outside the range covered by the baseline to cover possible extrapolation errors).

So far, the confidence intervals computed for different distances after Eq. (32) reflect solely measuring errors encountered in the field, or, in other words, repeatability or precision. This follows clearly from the composition of Eq. (5) in paragraph (12) of Section 3.2., for example. Although a scale correction had been applied to the reference distances, a constant bias would not affect the precisions obtained by the adjustment and thus the uncertainties of Eq. (32). Similarly, any constant errors in all temperatures and pressures measured would not affect the uncertainties as expressed by Eq. (32).

The uncertainty of the prescribed EDM equipment's scale correction Z_D, the calibration uncertainties Z_T of the thermometers and Z_B of the barometers and the uncertainty Z_E of the chosen yearly average partial water vapour pressure affect all the distance dependent terms of the instrument correction. Assuming that Z_D, Z_B, Z_T all reflect the uncertainty of the respective calibration constant against the respective National Standard at the 99% confidence level, a combined distance dependent term Z may be derived:

$$Z^2 = Z_D^2 + 0.25\ (Z_{T_1}^2 + Z_{T_2}^2) + 0.25\ (Z_{B_1}^2 + Z_{B_1}^2)\ (0.3)^2 + (0.04)^2\ Z_E^2 + Z_{T_3}^2 + (0.3)^2 Z_{B_3}^2 + Z_p^2 \quad (33)$$

where Z_{T_1} , Z_{T_2} = calibration uncertainty of two thermometers used in measuring the baseline with the certified EDM instrument (in degrees Celsius).

Z_{B_1}, Z_{B_2} = calibration uncertainty of the two barometers used in connection with the measuring of the baseline with the certified EDM instrument (in milibar).

and where Z, Z_D, Z_p are given in parts per million (ppm).

The calibration uncertainty of the thermometer used during the testing of the surveyor's EDM instrument is denoted by Z_{T_3} (in degree Celsius). This value may be taken from a test report of the thermometer concerned (e.g. NATA certificate). If the calibration is carried out by the surveyor himself against a Bureau of Meteorology weather station 'dry bulb' thermometer, this uncertainty should be based on the reading uncertainty on reference and surveyor's thermometer as well as on the accuracy of the weather station thermometer. The latter is given by the Australian Standard AS R13-1966 as ± 0.15° C.

Similarly, the calibration uncertainty of the barometer used during the testing of the surveyor's EDM instrument is denoted by Z_{B_3} (in millibar). this value may be taken from a test certificate of said barometer. Alternatively, the surveyor may calibrate his barometer against a mercury column barometer of a weather station of the Bureau of Meteorology. In such a case, the value Z_{B_3} should include the reading uncertainties on reference and surveyor's barometer as well as the accuracy of the weather station barometer. The latter may be taken as ± 0.4mb . (See Appendix B).

The uncertainty Z_E (in millibar) of the selected yearly mean partial water vapour pressure may be estimated on the basis of the Richmond N.S.W. data mentioned in paragraph (4), Section 2.1. From a histogramme of this 3358 day data set, the uncertainty at the 99% confidence level may be derived as:

$$Z_E = \pm 12.7 \text{ mb} \qquad (33a)$$

The uncertainty Z_p has to be considered whenever, on sloping baselines, pressures are measured at one terminal of lines only and whenever the pressure gradient with height is ignored. In the case of SPRENT/ZWART baselines and measurement of pressures at one end of the baseline only, this uncertainty Z_p (in parts per million) may be estimated as:

$$Z_p = 0.5\ (0.3)\ 0.120\ (\Delta H)$$

$$Z_p = \pm 0.018\ \Delta H \qquad (33b)$$

where ΔH is the difference in elevation (in metres) between the endpoints of the baseline.

The uncertainty of the instrument correction I.C. against the National Standard therefore becomes:

$$P\ (I.C. - q < \overline{I.C.} < I.C. + q) = 0.99 \qquad (34)$$

with

$$q = \left(\{t_{99\%,\ n-u}\ Q_{FF}^{0.5}\}^2 + Z^2 \{\frac{D}{1000}\}^2 \right)^{0.5} \qquad (35)$$

where q and I.C. in millimetre, Z in ppm and D in metre. If:

$$q \leq (3mm + 30ppm) \qquad (36)$$

for the shortest and the longest distance measured with the EDM instrument on the baseline, then the calibrated instrument fulfils the "minimum standard for the uncertainty of calibration of an EDM instrument" in terms of Recommendation No. 8 of the Working Party of the National Standard Commission "on the Calibration of E.D.M. Equipment" of 1 February, 1983.

4. NUMERICAL EXAMPLES

For the two baseline designs most common is Australia, one numerical example each is given. Real data are used in both cases. No attempt of comparing the two baseline designs can be made or should be made on the basis of these two examples because the accuracy of the data as such is not the same.

4.1. "MITCHAM" 'SPRENT-ZWART' BASELINE

The recent data sets for this baseline have been kindly provided by the Melbourne Metropolitan Board of Works. The baseline is of the " Sprent-Zwart Design", with a total length of 600m. The reference distances were measured with a Kern Mekometer ME 3000 in "all combinations". Of these 28 observations, three distances were not considered in the adjustment (two 5m, one 10m) because they failed in the data snooping phase. (I was advised subsequently that these three distances were measured by steel tape rather than Mekometer!). An instrument correction after Eq. (11) was solved for. The computer analysis may be found in Appendix C. The result is as follows:

$$\text{I.C. (1)} = -0.70 - 2.8 \left(\frac{D}{1000}\right) - 0.15 \sin \left(\frac{2\pi D}{10}\right) + 0.02 \cos \left(\frac{2\pi D}{10}\right) + 0.49 \sin \left(\frac{2\pi D}{5}\right) - 0.72 \cos \left(\frac{2\pi D}{5}\right) \quad \ldots (37)$$

where I.C. (1) in millimetre.

With a degree of freedom of 21, and for a two-tailed test

$$t_{(95\%,21)} = 2.08 \quad (38)$$

The test statistic (after Eq. (27)) for the largest cyclic error term yields

$$T = 0.72\ (0.21)^{-0.5} = 1.57 \quad (39)$$

The null hypothesis is therefore accepted and none of the cyclic error terms differs significantly from zero. (As mentioned before, the analysis should now be repeated without solving for short periodic errors).

Using Eqs. (29), (30) and

$$t_{99\%,21} = 2.83 \quad (40)$$

and the relevant submatrix of the "Variance Covariance Matrix of Adjusted Parameters" ($col._8 = a_o$, $col._9 = a_1$, $col._{12} = b_{12}$, $col._{14} = b_{22}$) of Appendix C, the following confidence limits for the instrument correction I.C., in terms of precision, were obtained.

Distance	$t_{99,21}\ (Q_{FF})^{0.5}$	Comments
0 m	(± 2.63 mm)	
70 m	± 2.42 mm	shortest distance
260 m	± 2.01 mm	mean distance
600 m	± 2.63 mm	longest distance
1200 m	(± 4.79 mm)	
1800 m	(± 7.70 mm)	
2400 m	(±10.73 mm)	

(41)

The 99% confidence intervals of calibration constants (in terms of National Standards) of the equipment involved in the certification measurements, namely of Mekometer, thermometer and barometers, respectively, are given as:

$$Z_D = \pm\ 5.0\ \text{ppm}$$

$$Z_{T_1} = Z_{T_1} = \pm\ 0.5^{\circ}\ C$$

$$Z_{B_1} = Z_{B_2} = \pm\ 1.0\ \text{mb} \qquad (42a)$$

The thermometer used during the measurements with the EDM to be tested was calibrated against a weather station 'dry bulb' thermometer, as outlined in Section 2.4.2. The mean calibration constant yield a standard deviation of ± 0.15° C. Considering

$$t_{99\%,3} = 3.18 \quad \text{(two-tailed test)}$$

and the uncertainty of the reference thermometer of ± 0.15°C (after AS R13-1966) the uncertainty of the calibration of the surveyor's thermometer Z_{T_3} may be obtained as

$$Z_{T_3}^2 = (\ 3.18\ \{0.15\}\ )^2 + (0.15)^2$$

$$Z_{T_3} = \pm\ 0.50^{\circ}\ C \qquad (42b)$$

The aneroid barometer used by the surveyor was calibrated against the mercury column barometer of the same weather station. Following a similar approach as above the uncertainty (at the 99% confidence level) of the calibration constant of the aneroid barometer Z_{B_3} yields:

$$Z_{B_3}^2 = (3.18\ \{0.06\}\ )^2 + (3.18\ \{0.11\})^2 + (0.4)^2$$

$$= \pm\ 0.5\ \text{mb} \qquad (42c)$$

where the first, second and third term reflect the uncertainties of the mercury barometer reading, the aneroid barometer reading and the mercury barometer as such, respectively. For the latter the tolerances given by the Bureau of Meteorology (see Appendix B) have been adopted.

The value for the uncertainty of the partial water vapour pressure Z_E is taken from Eq. (33a). As the pressures have been read only at one end of the baseline during the test of the surveyor's EDM instrument, and as the pressure gradient is not taken into account by computation, an additional uncertainty after Eq. (33b) must be considered. With the total elevation difference between the endpoints of the baseline of 15.5m:

$$Zp = \pm\ 0.018\ (15.5) = \pm\ 0.28\ \text{ppm} \qquad (42d)$$

Applying Eq. (33) yields

$$Z = \pm\ 5.08\ \text{ppm} \qquad (43)$$

Using Table (41) and applying Eqs. (34) and (35) yields

Distance	q	Recommendation No. 8
70 m	± 2.44 mm	± 5.10 mm
260 m	± 2.41 mm	±10.80 mm
600 m	± 4.03 mm	±21.00 mm
1200 m	(± 7.75 mm)	±39.00 mm
1800 m	(±11.95 mm)	±57.00 mm
2400 m	(±16.24 mm)	±75.00 mm

(44)

It follows from Table (44) that the instrument correction of the distance meter tested (see Eq. (37)) has an uncertainty which is in compliance with the stated Recommendation No. 8. The uncertainties of calibration values given in Eq. (42a) were however assumed and may be optimistic. Previous studies executed by the author in fact indicate that the (99% confidence level) uncertainty of Mekometer scale calibrations by frequency measurement is ± 7.7 ppm rather than ± 5.0 ppm. Sophisticated calibration procedures for the Mekometer may however reduce that value at some time in the future.

4.2. "EAGLE FARM" ('SCHWENDENER') BASELINE

The two data sets of this baseline have been kindly supplied by the Queensland Department of Mapping and Surveying at some time in the past. The baseline is of the original "Schwendener" Design with a total length of 1021 m. Both data sets of (21 distances each) were measured in "all combinations". Because no Mekometer data set was available (to the author), the data set gathered with a Hewlett Packard HP3808A distance meter was taken as reference.

An instrument correction following Eq. (11) was solved for. The computer analysis may be found in Appendix D. The result yields

$$\text{I.C. (1)} = + 135.84 - 7.0 \left(\frac{D}{1000}\right) + 1.67 \sin\left(\frac{2\pi D}{10}\right) + 1.88 \cos\left(\frac{2\pi D}{10}\right) + 0.48 \sin\left(\frac{2\pi D}{5}\right) + 0.62 \cos\left(\frac{2\pi D}{5}\right) \quad \ldots (45)$$

where I.C. (1) in millimetre.

Considering the degree of freedom of 29 and a two-tailed test, the critical t value yields

$$t_{(95\%,29))} = 2.05 \quad \ldots (46)$$

The test statistic (after Eq. (27)) for the largest cyclic error component (1.88 mm) becomes

$$T = 1.88\ (0.38)^{-0.5} = 3.05 \quad \ldots (47)$$

The 10 m - cosine term is therefore significantly different from zero at the 95% confidence level. The two 5 m terms are clearly not significant. The test statistic for the 10 m sine term is just below the critical value. It can therefore be concluded that the first order short periodic error of 10 m wavelength is significant and should be accounted for. (At this stage, the computer analysis should be repeated, without solving for a 5 m "cyclic" error).

Using Eqs. (29) and (30) as the critical value for a two-tailed test

$$t_{99\%,29} = 2.76 \quad \ldots (48)$$

and considering the relevant submatrix of the "Variance Covariance Matrix of Adjusted Parameters" of Appendix D (Col./line 7 = a_0 , Col./line 8 = a_1 , Col/line 11 = b_{12} , Col./line 13 = b_{22}), the confidence limits of the instrument correction as listed in Eq. (45) yield:

Distance	$t_{99\%,\ 29}(Q_{FF})^{0.5}$	Comments
20 m	± 3.32 mm	shortest distance
400 m	± 2.98 mm	mean (of all) distance
510 m	± 3.17 mm	
760 m	± 3.89 mm	
1020 m	± 4.94 mm	longest distance
2040 m	(± 9.95 mm)	
3060 m	(±15.37 mm)	
4080 m	(±20.87 mm)	

(49)

The necessarily fictitiuous uncertainties (at 99% confidence level) of the calibration parameters of the reference distance meter as well as thermometers and barometers (against National Standards) used during the reference measurement of the baseline are taken from Eq. (42a) for simplicity. Also, the same Z_{T_3}, Z_{B_3} and Z_E values are used as in Section 4.1 (Refer to Eqs. (42B) and (42c)). Using Eq. (33b) and a total height difference of 2.00m for the Eagle Farm baseline, the uncertainty Z_p, caused by measuring atmospheric pressures at one terminal of lines only, yields

$$Zp = \pm\ 0.04 \text{ ppm} \tag{50}$$

Application of Eq. (33) leads to:

$$Z = \pm\ 5.07 \text{ ppm} \tag{51}$$

Using Table (49) and Eqs. (34) and (35) leads to the uncertainty of the instrument correction of the tested distance meter (K & E Rangemaster II) against National Standards.

Distance	q	Recommendation No. 8
20 m	± 3.32 mm	± 3.6 mm
400 m	± 3.59 mm	±15.0 mm
1020 m	± 7.11 mm	±33.6 mm
2040 m	(±14.35 mm)	± 64.2 mm
3060 m	(±21.84 mm)	± 94.8 mm
4080 m	(±29.38 mm)	±125.4 mm

(52)

The uncertainty of the instrument correction of the distance meter tested complies therefore, within the tested distance range 20 m - 1020m, with the "minimum standard for the uncertainty of calibration of an EDM instrument" after Recommendation No. 8, Working Party of the National Standards Commission on the "Calibration of EDM Equipment" (1 February 1983).

5. ISSUE OF CERTIFICATES

The chairman of the working party of the National Standards Commission on the "Calibration of EDM Equipment" stated that formal certificates pursuant to Regulation 80, of the Weights and Measures (National Standards) Regulation, 1966, (so-called'Reg.80 certificates') may be issued by the verifying authorities for any (electro-optical short-range) distance meters provided that the measurements with this distance meter on the baseline were executed by the verifying authority itself. The Surveyor Generals as verifying authorities in respect of length may however decide to issue other types of "certificates" under state laws or regulations. In both cases the author suggests that the following items are mentioned in the "certificate" of one type or another:

(1) the date and times on which the EDM instrument and the reflector were verified or reverified;

(2) a description of the marks which have been legibly and permanently affixed to the EDM instrument and the reflector and, if applicable, to the theodolite and prism carrier.

(3) the period within which the EDM instrument is to be verified or reverified again;

(4) name of EDM calibration baseline and date of issue of its most recent Regulation 80 Certificate;

(5) the mathematical expression for the instrument correction as determined, clearly specifying the units of all parameters;

(6) the range of distances over which the above instrument correction has been verified, namely from the shortest to the longest line as measured on the baseline;

(7) the distance range, over which the periodic error component(s) of the instrument correction was determined;

(8) a list of the thermometer (s) and barometer (s) used during the calibration measurements, including a description of the marks which have been legibly and permanently affixed to these instruments;

(9) a list of the additive constants of the thermometer (s) and the barometer (s) including where and when these constants were determined and their uncertainties of one part in one hundred;

(10) a general description of the weather condition experienced during the tests including cloud cover, sun exposure, precipitation, wind;

(11) the range of temperatures experienced during the baseline measurements, and the mean temperature, the latter being the arithmetic mean of all temperature measurements taken;

(12) a statement saying if all measurements were executed during daytime or night time;

(13) a statement giving a description how the measurement procedures differed from the set guidelines (or noting that the guidelines have been followed);

(14) the value of any additive constant applied to all measurements prior to the least-square analysis;

(15) the value of any switchable or hardwired additive constant built-into the EDM instrument at the time of the calibration;

Note: It is evident that the a_0 term of the I.C. determined gives the increment to the values stated in (14) and/or (15) only.

(16) a clear statement that any term of the instrument correction being linear with distance refers to the mean temperature computed in paragraph (11) above; the stated instrument correction is therefore restricted to a distance range and one particular ambient temperature;

Note 1: The change of the distance dependent term of the instrument correction, namely the scale correction, can be easily determined by frequency calibration over a desired temperature range. The alternative is to determine the instrument correction at a low, a middle and a high ambient air temperature on a certified baseline.

Note 2: Until such time when scale corrections values for different temperatures become available, the surveyor has to rely on the manufacturer's 'accuracy' specifications. The ppm term stated by manufacturers is usually equivalent to the maximal change of the instrument's scale from the scale at 20°C to the upper and lower limits of operating temperatures stated.

(17) a clear statement that scale correction term refers to a setting of the 'ppm-dial' to zero ppm (or neutral) position and application of the first velocity correction by computation (see also (21));

(18) the uncertainty of one part in one hundred of the instrument correction listed in (5) for the shortest, mean and longest distance measured;

(19) the uncertainty of one part in one hundred of the instrument correction listed in (5) for the double, triple and quadruple of the longest baseline distance, clearly stating that this is based on extrapolation and should be taken as a guide only;

(20) whether or not the uncertainty of the instrument correction fulfils the minimum requirements of Recommendation No. 8, NSC Working Party on the "Calibration of EDM Equipment" from 1 February 1983;

(21) the first velocity correction used in the computations;

(22) a reference to the file where the original measurements and computations are kept, for future reference;

(23) any special comments the verifying authority wishes to make;

(24) a reference to a full description of the baseline used;

(25) in the case of EDM instruments mounted on the standards of a theodelite it should be stated whether the EDM instrument was attached in the 'face left' or 'face right' position (see paragraph (18) of Section 2.1);

(26) the owner of the EDM instrument and the reflector;

(27) the members of the survey party involved in the field measurement;

(28) the name and signature of the computing and verifying authority;

(29) the standard deviation (a posteriori) of a distance measurement (mean of four) executed with the distance meter on the baseline;

A sample test report is given in Appendix 'A'.

6. CONCLUDING REMARKS

It should be noted again that these (revised) instructions list the minimal procedures which should be followed by all surveyors, when calibrating their EDM instrument (s) against a certified baseline in fulfilment of requirements of State (or Territory) laws, by-laws, acts, regulations,... . Some optional procedures which are likely to produce more precise instrument corrections, are listed.

It is anticipated that most EDM instruments held by surveyors can be calibrated within the bounds of 'Recommendation No. 8' on any of the three baseline designs available in Australia, if the above instructions are followed.

Naturally, any further comments, criticisms and alternative proposals are welcome. They should preferably be addressed to the author with copy to:

The Secretary,
Recess Committee,
Reciprocating Surveyors Boards of Australian and
New Zealand,
P.O. Box 2,
BELCONNEN, A.C.T. 2616

(or vice-versa). When proposing alternative mathematical and/or statistical analyses, a fully worked numerical example (based on data given in Appendices C and D) would be appreciated.

Dr. Jean M. Rüeger,
9 April, 1984.

7. REFERENCE

RÜEGER, J.M. 1982: Introduction to Electronic Distance Measurement 2nd ed., 2nd imp., School of Surveying, University of New South Wales, 128 pages.

8. DISTRIBUTION

* The Secretary, Recess Committee, Reciprocating Surveyors Boards of Australia and New Zealand, P.O. Box 2, BELCONNEN, A.C.T. 2616.

* Mr. H.J. O'Meara, Australian Survey Office, P.O. Box 2, BELCONNEN, A.C.T. 2616.

* Mr. S.Gulbin, Survey Co-ordination Branch, Division of Survey and Mapping, 2 Treasury Place, MELBOURNE, Vic. 3002.

* Dr. D.L. Larden, R & D Section, Dept. of Lands, G.P.O. Box 1047, ADELAIDE S.A. 5001.

* Mr. F.H. Bray, Office of the Surveyor General, Cathedral Avenue, PERTH W.A. 6000.

* Mr. J. Veal, Dept. of Lands, G.P.O. Box 1680, DARWIN NT 5794.

* Mr. L.J. Glass, Dept. of Mapping and Surveying, P.O. Box 234, BRISBANE NORTH QUAY Qld.4000.

* Mr. C.C.A. Butler, Surveyor-General, Lands Dept., G.P.O. Box 44A, HOBART TAS.7001.

* Mr. G. Helsham, Survey Co-ordination Branch, Crown Lands Office, G.P.O. Box 39, SYDNEY N.S.W. 2001.

* Dr. A. Sprent, School of Surveying, University of Tasmania, G.P.O. Box 252C, HOBART TAS.7001.

* Mr. P.E. Ciddor, National Measurement Laboratory, CSIRO, P.O. Box 218, LINDFIELD N.S.W. 2070.

* Dr. H.M.P. Stock, Standards Branch, Dept. of Public and Consumers Affairs, G.P.O. Box 1268, ADELAIDE S.A. 5001.

* Mr. B. Murnane, Survey Division, Melbourne Metropolitan Board of Works, G.P.O. Box 4342, MELBOURNE, Vic. 3001.

APPENDIX A: SAMPLE TEST REPORT

SURVEYOR GENERAL OF
XXXXXX

TEST NO: 1248 DATE: 15 Oct 1983

S T A T E M E N T O F T E S T

ELECTRONIC DISTANCE METER

The EDM instrument AGA Geodimeter 112, serial number 23620, as mounted on the telescope of the theodolite WILD T 2, serial number 234618, together with AGA prism (AGA PART No. 571 125 021), as mounted in the centre hole of a tiltable AGA target (AGA Part No. 571 125 026), was calibrated on the MITCHAM Baseline in Melbourne on 10 October 1983 between 11.00 h and 12.00 h. The prism and the tiltable target carry the engraved marks "M.M.B.W. 427" and "M.M.B.W. 286", respectively.

The MITCHAM baseline was last measured and certified under Regulation 80 of the National Standards Regulations on 5 October 1983. A full description of this baseline may be found in the publication 'The Mitcham Baseline', as published by this Department (Publ. No.347, February 1983). The EDM instrument as well as the target/reflector are owned by the Melbourne Metropolitan Board of Works, Mitcham Area Office.

The measurements were executed in daylight and as specified in 'Recommended Field Procedures for...' (published by this Department, Publ. No. 286, March 1983). The reflector/target assembly was always pointed to the telescope of the theodolite using the gun sight in the centre of the target, both, horizontally and vertically. All observations were made on a sunny day with light NW winds. The temperatures varied from + 17° C to + 21° C, with a mean temperature of + 19.0° C.

The following first velocity correction was applied to all measurements by computation:

$$K' = \left(275.0 - \frac{79.6 \times P}{273.15 + t} + \frac{11.20 \times e}{273.15 + t}\right) 10^{-6} \times D$$

where t in °C, P and e in millibar. The yearly average partial water vapour pressure of 12.8 mb was used, as specified for the Mitcham Baseline.

No additive constant was applied to the measurements. Any built-in additive constants are unknown and inaccessible to the user. The observations were executed by Messrs. B. Green, P. Brown and G. Blue of the Melbourne Metropolitan Board of Works.

For the temperature measurement, a ZEAL mercury pocket thermometer, graduated from 0° C to 60° C at 1° C intervals, was used. The calibration constant of this thermometer was determined by comparison with the weather station dry bulb thermometer at Mitcham Post Office on 9 October 1983 and yielded + 0.4° C with an uncertainty of one part in one hundred of ± 0.5° C. This thermometer carries the engraved mark ' M.M.B.W. 846'. A THOMMEN pocket barometer "Everest 6000m' with the serial number 416452 was used for measurement of atmospheric pressure. Its calibration constant was derived by comparison with the mercury column barometer at the Mitcham Post Office on 9 October 1983 and yielded -16.4 mb with an uncertainty at one part in one hundred of ± 0.5 mb. Both calibrations were executed as specified by the 'Recommended Field Procedures for ...' published by this Department, Publ. No. 286, March 1983).

The instrument correction I.C. (in millimetre) was determined according to the 'Recommended Analysis Procedures...' (published by this Department, Publ. No. 287, March 1983) as follows:

$$I.C. = -0.7 - 2.8 \left(\frac{D}{1000}\right)$$

where D is the distance in metre. This instrument correction is valid in the distance range from 70 m to 600 m and for an ambient temperature of +19.0° C. The second term of the instrument correction refers to a setting of the ppm-dial to ±0 ppm and use of the first velocity correction listed above.

The uncertainty of one part in one hundred of the above instrument correction I.C. is:

at 70 m	±2.44 mm
at 260 m	±2.40 mm
at 600 m	±4.00 mm

As a guide only, the uncertainties of the instrument correction are also given for longer distances. (When considering these values, due allowance should be made for the fact that they are based on extrapolation.)

at 1200 m	(± 7.8 mm)
at 1800 m	(±12.0 mm)
at 2400 m	(±16.2 mm)

This instrument/reflector set fulfils the requirements of Recommendation No. 8 of the NSC Working Party on the "Calibration of EDM Equipment" (Meeting of 1 February 1983).

The periodic error was tested over a wavelength of 10 m at distances ranging from 70 m to 600 m; it was found to be insignificant. The measured distances (mean of four) were found to have a precision of ± (0.6mm + 0.4ppm).

The original measurements and computations may be found in file 'TEST 1248' of this Department. The instrument should be reverified on or before 5 October 1984.

Certified on the fifteenth day
of October 1983

For the Surveyor General of
XXXXXXX

A.B.C. Black

APPENDIX B: BUREAU OF METEOROLOGY, REGIONAL OFFICE ADDRESSES AND LETTER RE STATION BAROMETERS

A.C.T.
Regional Director
Canberra City Office
Bureau of Meteorology
P.O. Box 797
Canberra City, A.C.T. 2601
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Regional Director
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Bureau of Meteorology
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NORTHERN TERRITORY
Regional Director
N.T. Regional Office
Bureau of Meteorology
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QUEENSLAND
Regional Director
Qld Regional Office
Bureau of Meteorology
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Tel:(07) 225 2766

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TASMANIA
Regional Director
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VICTORIA
Regional Director
Vic.Regional Office
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Tel:(03) 662 2555

WESTERN AUSTRALIA
Regional Director
W.A. Regional Office
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BUREAU OF METEOROLOGY

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AUSTRALIA

Telephone (03) 662 0311
Telex 30664 (METAUST) AA
Telegrams METAUST MELBOURNE

In reply please quote 25/2174 Please mark your reply Attention

Dr J.M. Rueger,
School of Surveying
University of New
South Wales,
P.O. Box 1,
KENSINGTON, NSW, 2033

9 APR 1984

Dear Dr Rueger,

ACCURACY OF KEW STATION BAROMETERS

I refer to your letter of 23 March 1984 regarding the accuracy of mercury Kew station barometers used by the Bureau of Meteorology.

The Bureau of Meteorology maintains a primary standard barometer which is the meteorological standard for the Regional Association V (ie South-West Pacific area) of the World Meteorological Organisation. This barometer is also the standard to which all barometers used by the Bureau are kept traceable. The Bureau's laboratories are not NATA registered, but the standard barometer is routinely calibrated against standards held by the National Measurements Laboratory.

Kew barometers specially selected for use as working standards are held by our Regional Offices to provide traceability for the transfer standard barometers used by inspectors. These working standards are routinely compared to the Head Office standard by means of transfer standards, and their readings have an accuracy of within about ± 0.2mb. The working standards are issued with detailed correction tables which include index corrections obtained over their working range and corrections to standard gravity and temperature conditions. The tables are used to obtain station level pressures.

Barometers issued to field stations have been calibrated over their working range and are within performance tolerances specified by BS2520 (NB. No equivalent Australian Standard). The accuracy of calibration is ± 0.2mb, but each barometer is checked at ambient pressures against working standards up to the time of issue, so the average index correction is

.../2

probably correct to within about ± 0.1mb. Calibration certificates are not issued with barometers. An index correction appropriate to the average station level pressure at each field site is used for the preparation of a correction table to be issued with a barometer.

It should be noted that small variations in temperature distribution within a barometer, the effects of rising and falling pressures on the meniscus of the barometer, observer error (eg. parallax error in readings), etc all contribute to a spread of about ± 0.15mb in readings. Even if the average index correction is known to within about ± 0.1mb, individual readings may, as a result, vary from the true pressure by up to ± 0.2mb. The accuracy of comparison with another instrument (eg. a surveyor's barometer) will be a function of the accuracy of the mercury barometer, the accuracy of the compared instrument, and the different responses to changing conditions (eg. wind and temperature) at the time of the comparison.

Bureau inspectors check the index corrections of Kew station barometers by means of transfer standard barometers when they visit stations. The accuracy of the check (related to Head Office standards) is about ± 0.3mb. Any field barometer having an apparent index error of greater than ± 0.5mb is replaced. Such measures ensure that field barometers have an accuracy of about ± 0.2mb when issued, but are not expected to have an average accuracy of worse than ± 0.4mb even after a long period of use at stations. Individual readings of a field Kew barometer are therefore expected to be within ± 0.5mb of the true atmospheric pressures.

Correction tables issued with field barometers include corrections to standard gravity and temperature conditions. The gravity correction value and the index correction value appropriate to the average station level pressure are both stated on the tables, but are not applied to observations as they are already included in table values. The tables are used to obtain both station and sea-level pressures.

I trust this information satisfactorily answers your queries.

Yours sincerely,

(A.F. YOUNG)
<u>for Director of Meteorology</u>

APPENDIX: ANALYSIS OF A 'SPRENT-ZWART' BASELINE DATA SET

```
                         CALIBRATION OF EDM INSTRUMENT
              PROGRAM DEVELOPED FROM EDMCAL (J.D. LOVE) BY P.C.COVELL UNSW, 1980
                         REVISED BY J.M. RUEGER: 9.1.1984
INSTRUMENT  AGA 112 (23620)&ME3000(218106)  BASELINE USED  M.M.B.W. 'MITCHAM', MELBOURNE   DATE OF OBSERVATIONS  5 & 10 OCTOBER 198
                         COMPUTED FOR  J.M. RUEGGER
          NUMBER OF PILLARS ON BASE LINE 8        NUMBER OF DISTANCES OBSERVED MODE 1&2 =10   MODE 3 =25
              STANDARD DEVIATION (A PRIORI) MODE 1 =    .70MM +    .50PPM
                                            MODE 2 =   0.00MM +   0.00PPM
                                            MODE 3 =    .30MM +    .50PPM
IFLAG1 =  0        IFLAG2 =  0
CURVE 1     10.00        CURVE 2      5.00
     ELEVATIONS OF PILLARS USED (METRES)
          PILLAR 1  15.525    PILLAR 2  15.328    PILLAR 3  15.177    PILLAR 4  12.956
          PILLAR 5  10.534    PILLAR 6   6.764    PILLAR 7   3.695    PILLAR 8   0.000
     HEIGHT OF EDM ABOVE PILLAR PLATE = .236METRES
     HEIGHT OF REFLECTOR ABOVE PILLAR PLATE =  .143 METRES
     HEIGHT OF EDM ABOVE PILLAR PLATE = .260METRES
     HEIGHT OF REFLECTOR ABOVE PILLAR PLATE =  .260 METRES
```

SLOPE REDUCTIONS AND STANDARD DEVIATIONS(A PRIORI) OF DISTANCES

OCCUPIED PILLAR	OBSERVED PILLAR	SLOPE DISTANCE M	HORIZONTAL DISTANCE M	MODE USED	STD DEV OF DISTANCE MM
1	4	72.06350	72.01433	1	.74
1	5	134.07780	133.98143	1	.77
1	6	236.11880	235.95289	1	.82
1	7	378.22100	378.03336	1	.89
1	8	600.12320	599.92065	1	1.00
2	8	595.08280	594.88367	1	1.00
2	7	373.17850	372.99457	1	.89
2	6	231.07620	230.91414	1	.82
2	5	129.03580	128.94327	1	.76
2	4	67.01980	66.97447	1	.73

1	4	72.05825	72.01245	3	.34
1	5	134.07117	133.97829	3	.37
1	6	236.11314	235.95070	3	.42
1	7	378.21605	378.03133	3	.49
1	8	600.11751	599.91737	3	.60
2	4	67.01539	66.97341	3	.33
2	5	129.02890	128.93986	3	.36
2	6	231.07007	230.91147	3	.42
2	7	373.17200	372.99097	3	.49
2	8	595.07408	594.87735	3	.60
3	4	62.09891	62.05919	3	.33
3	5	124.11202	124.02519	3	.36
3	6	226.15348	225.99709	3	.41
3	7	368.25662	368.07791	3	.48
3	8	590.15876	589.96429	3	.60
4	5	62.01251	61.96523	3	.33
4	6	164.05397	163.93721	3	.38
4	7	306.15738	306.01761	3	.45
4	8	528.06150	527.90327	3	.56
5	6	102.04016	101.97060	3	.35
5	7	244.14596	244.05047	3	.42
5	8	466.05428	465.93595	3	.53
6	7	142.11129	142.07837	3	.37
6	8	364.02699	363.96482	3	.48
7	8	221.91467	221.88437	3	.41

SOLUTION TO NORMAL EQUATIONS

PILLARS 1 TO 2	DISTANCE = 5.03909M	STANDARD DEVIATION =	.23MM *
PILLARS 1 TO 3	DISTANCE = 9.95323M	STANDARD DEVIATION =	.25MM *
PILLARS 1 TO 4	DISTANCE = 72.01370M	STANDARD DEVIATION =	.23MM *
PILLARS 1 TO 5	DISTANCE = 133.98018M	STANDARD DEVIATION =	.26MM *
PILLARS 1 TO 6	DISTANCE = 235.95214M	STANDARD DEVIATION =	.29MM *
PILLARS 1 TO 7	DISTANCE = 378.03226M	STANDARD DEVIATION =	.33MM *
PILLARS 1 TO 8	DISTANCE = 599.91827M	STANDARD DEVIATION =	.39MM *
ADDITIVE CONSTANT MODE 1	= -.00070M	STANDARD DEVIATION =	.53MM *
ADDITIVE CONSTANT MODE 3	= .00143M	STANDARD DEVIATION =	.18MM *
SCALE FACTOR =	.9999972105	STANDARD DEVIATION=	1.79PPM *

CURVE M	SINE MM	COSINE MM	AMPLITUDE MM	S.D. OF AMP MM	PHASE DEG	S.D. OF PHASE DEG
10.	-.1466	.0174	.15	.36	173.2	141.2
5.	.4906	-.7218	.87	.41	304.2	26.8

* USING A POSTERIORI VARIANCE FACTOR = .962

T VALUE 2.060

STANDARD DEVIATION (APRIORI)	MODE 1=	.70MM	.50PPM
VARIANCE FACTOR =1.000	MODE 2=	0.00MM	0.00PPM
	MODE 3=	.30MM	.50PPM
STANDARD DEVIATION (POST)	MODE 1=	.69MM	.49PPM
VARIANCE FACTOR = .962	MODE 3=	.29MM	.49PPM
STANDARD DEVIATION (POST)	MODE 1&2 =	.59MM	.42PPM
VARIANCE FACTOR = .700			
STANDARD DEVIATION (POST)	MODE 3 =	.31MM	.52PPM
VARIANCE FACTOR =1.068			

RESULTS OF CALIBRATION AND DATA SNOOPING

ADDITIVE CONSTANT (MODE 1) = -.00070METRES
STANDARD DEVIATION (MODE 1) = .53MM
95% CONFIDENCE INTERVAL (MODE 1) FROM -.00180 TO .00040METRES

VARIANCE FACTORS -
A PRIORI.... 1.000
A POSTERIORI .962

MULTIDIMENSIONAL TEST ON VARIANCE FACTOR ESTIMATE (TEST 1 BAARDA)
TEST VALUE = .962
CRITICAL 'F' VALUE =1.290 (INVERSE = .775)

THE NULL HYPOTHESIS IS ACCEPTED

OCCUPIED PILLAR	OBSERVED PILLAR	MODE USED	HORIZONTAL DISTANCE M	ADJUSTED DISTANCE M $	STD DEV OF ADJ DIST MM $$	RESIDUAL V MM	STD DEV OF RESIDUAL (SV) MM $$$	TEST 2 V/SV $$$$	ALARM
1	4	1	72.01433	72.01343	.58	-.46	.44	1.05	
1	5	1	133.98143	133.98035	.65	.61	.38	1.59	
1	6	1	235.95289	235.95153	.60	.34	.54	.63	
1	7	1	378.03336	378.03161	.68	.24	.55	.44	
1	8	1	599.92065	599.91828	.73	.74	.67	1.11	
2	8	1	594.88367	594.88131	.73	-1.31	.67	1.97	
2	7	1	372.99457	372.99283	.68	.19	.55	.34	
2	6	1	230.91414	230.91279	.61	.17	.53	.33	
2	5	1	128.94327	128.94221	.65	-.57	.38	1.48	
2	4	1	66.97447	66.97358	.57	.03	.44	.08	

1	4	3	72.01245	72.01389	.21	-.18	.26	.72
1	5	3	133.97829	133.97972	.21	.46	.30	1.54
1	6	3	235.95070	235.95213	.21	.01	.36	.02
1	7	3	378.03133	378.03276	.24	-.50	.43	1.18
1	8	3	599.91737	599.91880	.28	-.53	.53	1.00
2	4	3	66.97341	66.97484	.21	-.23	.25	.90
2	5	3	128.93986	128.94129	.21	-.19	.30	.65
2	6	3	230.91147	230.91290	.21	.15	.35	.42
2	7	3	372.99097	372.99241	.23	.77	.42	1.81
2	8	3	594.87735	594.87878	.28	.41	.53	.78
3	4	3	62.05919	62.06062	.22	-.16	.25	.63
3	5	3	124.02519	124.02662	.21	.33	.29	1.13
3	6	3	225.99709	225.99853	.22	.38	.35	1.08
3	7	3	368.07791	368.07935	.24	-.32	.42	.77
3	8	3	589.96429	589.96572	.28	-.68	.52	1.31
4	5	3	61.96523	61.96666	.22	-.18	.24	.75
4	6	3	163.93721	163.93865	.20	-.21	.32	.66
4	7	3	306.01761	306.01905	.20	-.49	.40	1.22
4	8	3	527.90327	527.90470	.23	-.13	.51	.25
5	6	3	101.97060	101.97203	.23	-.08	.26	.30
5	7	3	244.05047	244.05190	.22	.18	.36	.49
5	8	3	465.93595	465.93738	.23	.71	.48	1.48
6	7	3	142.07837	142.07980	.24	.32	.28	1.14
6	8	3	363.96482	363.96626	.24	-.12	.42	.29
7	8	3	221.88437	221.88581	.26	.21	.32	.66

NOTES:-
$ ADDITIVE CONSTANT INCLUDED
$$ USING A POSTERIORI VARIANCE FACTOR
$$$ USING A PRIORI VARIANCE FACTOR
$$$$ ONE DIMENSIONAL TEST(TEST 2 BAARDA):
THE NULL HYPOTHESIS IS ACCEPTED AS LONG AS TEST VALUE V/SV
IS LESS THAN THE CRITICAL VALUE 3.29

VARIANCE COVARIANCE MATRIX OF ADJUSTED PARAMETERS
(USING A PRIORI VARIANCE FACTOR =1.00)= COFACTOR MATRIX OF ADJUSTED PARAMETERS = QXX

DIST 1-2	DIST 1-3	DIST 1-4	DIST 1-5	DIST 1-6	DIST 1-7	DIST 1-8	A.C. 1	A.C. 2
.55E-01	.28E-01	.28E-01	.28E-01	.28E-01	.28E-01	.28E-01	-.62E-03	.31E-02
.43E-04	.49E-02	-.50E-02	-.14E-04	-.36E-03				
.28E-01	.65E-01	.31E-01	.31E-01	.31E-01	.31E-01	.31E-01	.17E-01	.14E-02
-.15E-04	.25E-02	-.25E-02	-.57E-05	-.18E-03				
.28E-01	.31E-01	.54E-01	.44E-01	.49E-01	.54E-01	.60E-01	.30E-01	.29E-01
.20E-01	.25E-02	-.25E-02	.32E-02	-.45E-02				
.28E-01	.31E-01	.44E-01	.69E-01	.59E-01	.66E-01	.76E-01	.41E-01	.27E-01
.28E-01	.24E-02	-.25E-02	-.86E-02	.71E-02				
.28E-01	.31E-01	.49E-01	.59E-01	.87E-01	.79E-01	.91E-01	.39E-01	.72E-01
.36E-01	.24E-02	-.23E-02	.58E-02	.62E-02				
.28E-01	.31E-01	.54E-01	.66E-01	.79E-01	.11	.11	.32E-01	.14
.45E-01	.24E-02	-.24E-02	-.48E-02	-.10E-01				
.28E-01	.31E-01	.60E-01	.76E-01	.91E-01	.11	.16	.37E-01	.17
.55E-01	.24E-02	-.25E-02	-.31E-03	.61E-02				
-.62E-03	.17E-01	.30E-01	.41E-01	.39E-01	.32E-01	.37E-01	.29	-.76
.19E-01	.20E-02	.88E-03	-.24E-01	.11				
.31E-02	.14E-02	.29E-01	.27E-01	.72E-01	.14	.17	-.76	3.3
.64E-01	-.73E-02	-.28E-02	.10	-.39				
.43E-04	-.15E-04	.20E-01	.28E-01	.36E-01	.45E-01	.55E-01	.19E-01	.64E-01
.32E-01	-.42E-04	.59E-04	-.81E-03	.94E-03				
.49E-02	.25E-02	.25E-02	.24E-02	.24E-02	.24E-02	.24E-02	.20E-02	-.73E-02
-.42E-04	.13	.51E-03	.24E-03	.13E-02				
-.50E-02	-.25E-02	-.25E-02	-.25E-02	-.23E-02	-.24E-02	-.25E-02	.88E-03	-.28E-02
.59E-04	.51E-03	.15	.61E-03	.15E-03				
-.14E-04	-.57E-05	.32E-02	-.86E-02	.58E-02	-.48E-02	-.31E-03	-.24E-01	.10

-.81E-03	.24E-03	.61E-03	.14	-.71E-03				
-.36E-03	-.18E-03	-.45E-02	.71E-02	.62E-02	-.10E-01	.61E-02	.11	-.39
.94E-03	.13E-02	.15E-03	-.71E-03	.21				

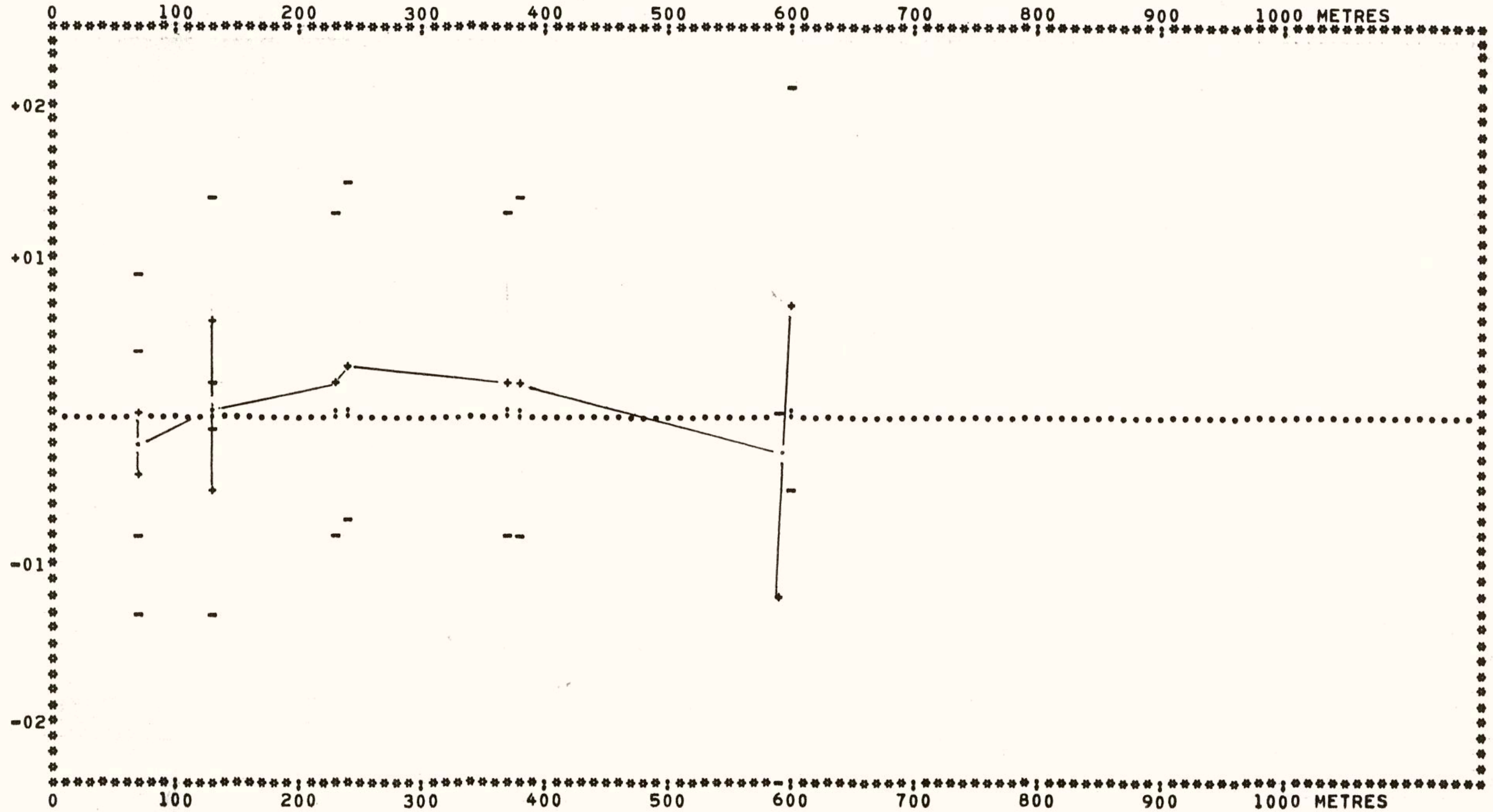
DIAGONAL ELEMENTS OF THIS MATRIX ARE :-

.55E-01	.65E-01	.54E-01	.69E-01	.87E-01	.11	.16	.29	3.3
.32E-01	.13	.15	.14	.21				

MODE 1

RESIDUALS (+) WITH 95% CONFIDENCE INTERVALS (-) (USING A POSTERIORI VARIANCE FACTOR) VERSUS DISTANCE

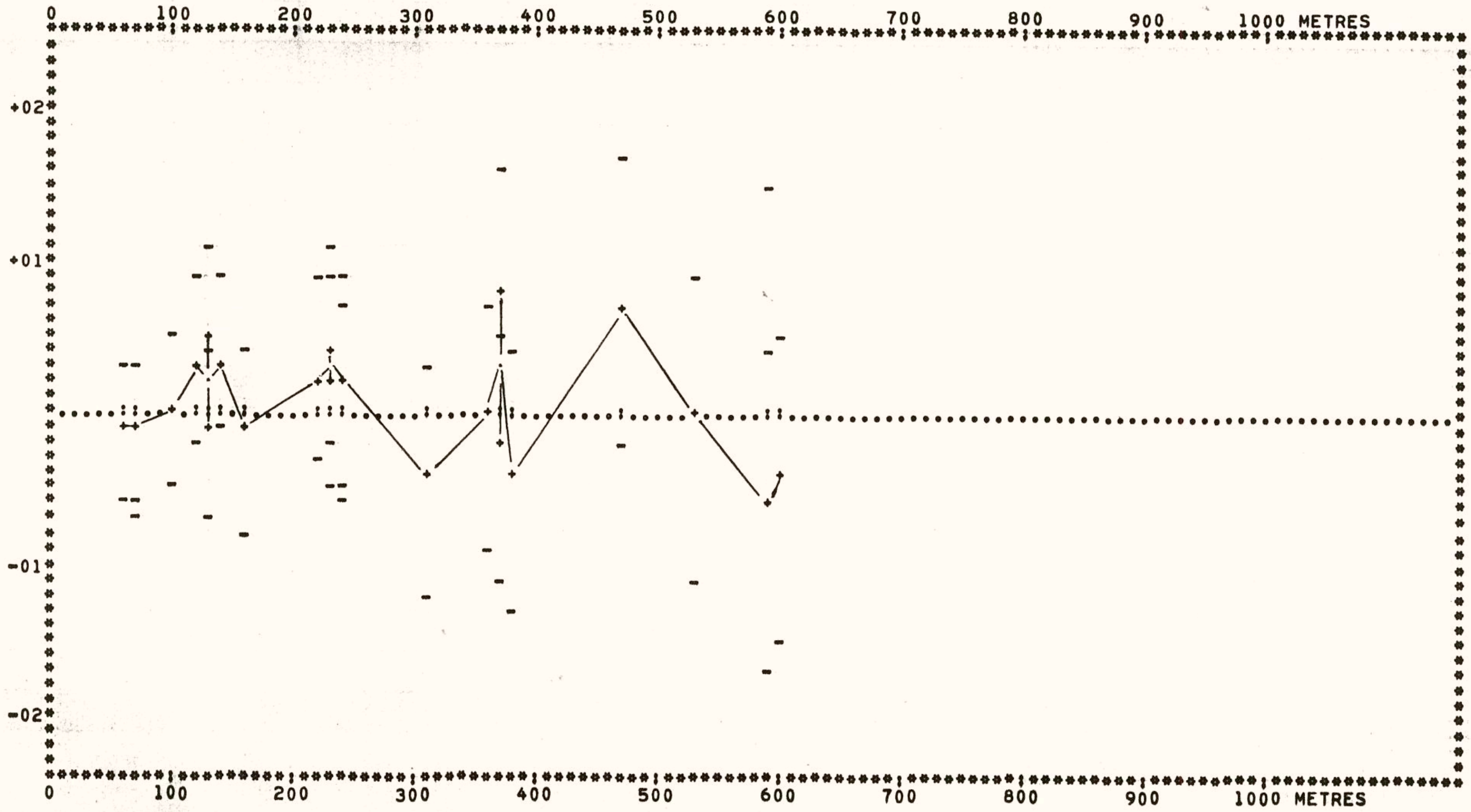
SCALES HORIZONTAL 1:4000 (APPROX)
VERTICAL.. 32.5:1 (APPROX)



MODE 3

RESIDUALS (+) WITH 95% CONFIDENCE INTERVALS (-) (USING A POSTERIORI VARIANCE FACTOR) VERSUS DISTANCE

SCALES HORIZONTAL 1:4000 (APPROX)
VERTICAL.. 32.5:1 (APPROX)



APPENDIX D: ANALYSIS OF A 'SCHWENDENER BASELINE DATA SET

CALIBRATION OF EDM INSTRUMENT
PROGRAM DEVELOPED FROM EDMCAL (J.D. LOVE) BY P.C.COVELL UNSW, 1980
REVISED BY J.M. RUEGER: 9.1.1984

INSTRUMENT K&E RANGEMASTER 2 & HP 3808 BASELINE USED EAGLE FARM, BRISBANE, QLD DATE OF OBSERVATIONS 12.7.79 & 5.11.79

COMPUTED FOR J.M. RUEGER

NUMBER OF PILLARS ON BASE LINE 7 NUMBER OF DISTANCES OBSERVED MODE 1&2 =21 MODE 3 =21

STANDARD DEVIATION (A PRIORI) MODE 1 = 1.80MM + .40PPM
MODE 2 = 0.00MM + 0.00PPM
MODE 3 = .65MM + .35PPM

IFLAG1 = 0 IFLAG2 = 0
CURVE 1 10.00 CURVE 2 5.00

ELEVATIONS OF PILLARS USED (METRES)

PILLAR 1	4.584	PILLAR 2	4.584	PILLAR 3	4.584	PILLAR 4	4.584
PILLAR 5	4.584	PILLAR 6	4.584	PILLAR 7	4.584	PILLAR 8	0.000

HEIGHT OF EDM ABOVE PILLAR PLATE = .100METRES
HEIGHT OF REFLECTOR ABOVE PILLAR PLATE = .100 METRES

HEIGHT OF EDM ABOVE PILLAR PLATE = .200METRES
HEIGHT OF REFLECTOR ABOVE PILLAR PLATE = .200 METRES

SLOPE REDUCTIONS AND STANDARD DEVIATIONS(A PRIORI) OF DISTANCES

OCCUPIED PILLAR	OBSERVED PILLAR	SLOPE DISTANCE M	HORIZONTAL DISTANCE M	MODE USED	STD DEV OF DISTANCE MM
1	2	511.37100	511.37099	1	2.00
1	3	767.37000	767.36999	1	2.11
1	4	894.90400	894.90399	1	2.16
1	5	962.89300	962.89298	1	2.19
1	6	1001.89200	1001.89198	1	2.20
1	7	1021.40500	1021.40498	1	2.21
2	3	255.86000	255.86000	1	1.90
2	4	383.39100	383.39099	1	1.95
2	5	451.37800	451.37799	1	1.98
2	6	490.38500	490.38499	1	2.00
2	7	509.89100	509.89099	1	2.00
3	4	127.40400	127.40400	1	1.85
3	5	195.38800	195.38800	1	1.88
3	6	234.39500	234.39500	1	1.89
3	7	253.90200	253.90200	1	1.90
4	5	67.85300	67.85300	1	1.83
4	6	106.85800	106.85800	1	1.84
4	7	126.36800	126.36800	1	1.85
5	6	38.87200	38.87200	1	1.82
5	7	58.38100	58.38100	1	1.82
6	7	19.36900	19.36900	1	1.81

1	2	511.50900	511.50898	3	.83
1	3	767.49900	767.49898	3	.92
1	4	895.03500	895.03497	3	.96
1	5	963.02100	963.02097	3	.99
1	6	1002.02700	1002.02697	3	1.00
1	7	1021.53400	1021.53397	3	1.01
2	3	255.99100	255.99099	3	.74
2	4	383.52700	383.52699	3	.78
2	5	451.51200	451.51199	3	.81
2	6	490.51900	490.51898	3	.82
2	7	510.02800	510.02798	3	.83
3	4	127.53400	127.53400	3	.69
3	5	195.52100	195.52099	3	.72
3	6	234.52800	234.52799	3	.73
3	7	254.03600	254.03599	3	.74
4	5	67.98800	67.98800	3	.67
4	6	106.99400	106.99400	3	.69
4	7	126.50100	126.50100	3	.69
5	6	39.00800	39.00800	3	.66
5	7	58.51600	58.51600	3	.67
6	7	19.50800	19.50800	3	.66

```
                                    SOLUTION TO NORMAL EQUATIONS
PILLARS 1 TO 2    DISTANCE =  511.50782M    STANDARD DEVIATION =     .45MM *
PILLARS 1 TO 3    DISTANCE =  767.49897M    STANDARD DEVIATION =     .49MM *
PILLARS 1 TO 4    DISTANCE =  895.03330M    STANDARD DEVIATION =     .54MM *
PILLARS 1 TO 5    DISTANCE =  963.01961M    STANDARD DEVIATION =     .60MM *
PILLARS 1 TO 6    DISTANCE = 1002.02658M    STANDARD DEVIATION =     .68MM *
PILLARS 1 TO 7    DISTANCE = 1021.53423M    STANDARD DEVIATION =     .76MM *
ADDITIVE CONSTANT MODE 1   =      .13584M    STANDARD DEVIATION =     .88MM *
ADDITIVE CONSTANT MODE 3   =    -.00067M    STANDARD DEVIATION =     .33MM *
SCALE FACTOR =    .9999929716           STANDARD DEVIATION=      1.96PPM *
CURVE      SINE      COSINE      AMPLITUDE      S.D. OF AMP      PHASE      S.D. OF PHASE
  M         MM         MM           MM               MM           DEG             DEG
  10.    1.6652     1.8813         2.51              .73          48.5            16.5
   5.     .4791      .6186          .78              .64          52.2            47.2

* USING A POSTERIORI VARIANCE FACTOR =   .991
          T VALUE   1.960
STANDARD DEVIATION (APRIORI) MODE 1=   1.80MM    .40PPM
VARIANCE FACTOR =1.000       MODE 2=   0.00MM   0.00PPM
                             MODE 3=    .65MM    .35PPM
STANDARD DEVIATION (POST)    MODE 1=   1.79MM    .40PPM
VARIANCE FACTOR = .991       MODE 3=    .65MM    .35PPM
STANDARD DEVIATION (POST) MODE 1&2  =   1.77MM    .39PPM
VARIANCE FACTOR = .964
STANDARD DEVIATION (POST) MODE  3   =    .66MM    .35PPM
VARIANCE FACTOR =1.017
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RESULTS OF CALIBRATION AND DATA SNOOPING

ADDITIVE CONSTANT (MODE 1) = .13584METRES
STANDARD DEVIATION (MODE 1) = .88MM
95% CONFIDENCE INTERVAL (MODE 1) FROM .13411 TO .13757METRES

VARIANCE FACTORS -
A PRIORI.... 1.000
A POSTERIORI .991

MULTIDIMENSIONAL TEST ON VARIANCE FACTOR ESTIMATE (TEST 1 BAARDA)
TEST VALUE = .991
CRITICAL 'F' VALUE =1.240 (INVERSE = .806)

THE NULL HYPOTHESIS IS ACCEPTED

OCCUPIED PILLAR	OBSERVED PILLAR	MODE USED	HORIZONTAL DISTANCE M	ADJUSTED DISTANCE M $	STD DEV OF ADJ DIST MM $$	RESIDUAL V MM	STD DEV OF RESIDUAL (SV) MM $$$	TEST 2 V/SV $$$$	ALARM
1	2	1	511.37099	511.50324	1.15	1.71	1.64	1.04	
1	3	1	767.36999	767.50043	1.37	.88	1.59	.55	
1	4	1	894.90399	895.03354	1.49	.99	1.55	.64	
1	5	1	962.89298	963.02206	1.25	-2.83	1.79	1.58	
1	6	1	1001.89198	1002.02078	1.10	3.66	1.90	1.93	
1	7	1	1021.40498	1021.53365	1.12	-2.25	1.90	1.18	
2	3	1	255.86000	255.99404	1.03	-1.13	1.60	.71	
2	4	1	383.39099	383.52414	1.17	1.64	1.56	1.05	
2	5	1	451.37799	451.51066	1.12	-1.73	1.63	1.06	
2	6	1	490.38499	490.51739	1.04	-1.62	1.70	.95	
2	7	1	509.89099	510.02325	1.12	.86	1.66	.52	
3	4	1	127.40400	127.53894	.98	-2.28	1.57	1.45	
3	5	1	195.38800	195.52246	1.03	-.36	1.57	.23	
3	6	1	234.39500	234.52919	1.02	-.57	1.59	.36	
3	7	1	253.90200	254.03605	1.09	-.04	1.56	.03	
4	5	1	67.85300	67.98836	.94	-.08	1.56	.05	
4	6	1	106.85800	106.99309	.94	2.54	1.58	1.61	
4	7	1	126.36800	126.50295	1.00	.09	1.55	.06	
5	6	1	38.87200	39.00757	.98	-.56	1.53	.37	
5	7	1	58.38100	58.51643	.91	-.68	1.58	.43	
6	7	1	19.36900	19.50470	1.05	1.76	1.47	1.20	

1	2	3	511.50898	511.50832	.49	-.50	.67	.75
1	3	3	767.49898	767.49831	.46	.66	.79	.83
1	4	3	895.03497	895.03431	.45	-1.00	.85	1.18
1	5	3	963.02097	963.02030	.46	-.69	.87	.79
1	6	3	1002.02697	1002.02630	.49	.28	.87	.32
1	7	3	1021.53397	1021.53330	.54	.93	.85	1.09
2	3	3	255.99099	255.99033	.45	.83	.59	1.41
2	4	3	383.52699	383.52632	.41	-.84	.67	1.25
2	5	3	451.51199	451.51132	.39	.47	.71	.67
2	6	3	490.51898	490.51832	.40	.44	.72	.62
2	7	3	510.02798	510.02732	.43	-.90	.70	1.28
3	4	3	127.53400	127.53333	.44	1.01	.54	1.87
3	5	3	195.52099	195.52033	.40	.32	.60	.53
3	6	3	234.52799	234.52733	.38	.28	.63	.45
3	7	3	254.03599	254.03533	.39	-.06	.63	.10
4	5	3	67.98800	67.98733	.43	-1.02	.52	1.98
4	6	3	106.99400	106.99333	.39	-.05	.57	.10
4	7	3	126.50100	126.50033	.37	.60	.59	1.02
5	6	3	39.00800	39.00733	.43	-.37	.51	.72
5	7	3	58.51600	58.51533	.39	-.71	.55	1.30
6	7	3	19.50800	19.50733	.42	.32	.50	.64

NOTES:-

$ ADDITIVE CONSTANT INCLUDED

$$ USING A POSTERIORI VARIANCE FACTOR

$$$ USING A PRIORI VARIANCE FACTOR

$$$$ ONE DIMENSIONAL TEST(TEST 2 BAARDA):
THE NULL HYPOTHESIS IS ACCEPTED AS LONG AS TEST VALUE V/SV
IS LESS THAN THE CRITICAL VALUE 3.29

VARIANCE COVARIANCE MATRIX OF ADJUSTED PARAMETERS
(USING A PRIORI VARIANCE FACTOR =1.00)= COFACTOR MATRIX OF ADJUSTED PARAMETERS = QXX

DIST 1-2	DIST 1-3	DIST 1-4	DIST 1-5	DIST 1-6	DIST 1-7	A.C. 1	A.C. 2	
.21	.15	.16	.17	.18	.19	-.26E-01	.19	.37E-01
-.11E-01	-.20E-02	-.37E-02	-.22E-01					
.15	.25	.20	.22	.24	.26	-.28E-01	.28	.68E-01
-.30E-01	.16E-01	.63E-02	-.40E-02					
.16	.20	.30	.26	.29	.32	.82E-02	.29	.10E+00
.69E-02	.44E-04	-.20E-01	.82E-02					
.17	.22	.26	.37	.34	.39	.50E-01	.27	.13
.21E-01	-.65E-02	.14E-01	-.24E-02					
.18	.24	.29	.34	.46	.45	.70E-01	.30	.16
.12E-01	.44E-02	.18E-02	.61E-02					
.19	.26	.32	.39	.45	.58	.99E-01	.32	.20
.12E-01	.20E-01	-.79E-02	.22E-01					
-.26E-01	-.28E-01	.82E-02	.50E-01	.70E-01	.99E-01	.78	-1.4	.54E-01
.45	.12E-01	.15	-.95E-01					
.19	.28	.29	.27	.30	.32	-1.4	3.9	.85E-01
-1.0	-.30E-01	-.41	.28					
.37E-01	.68E-01	.10E+00	.13	.16	.20	.54E-01	.85E-01	.11
.13E-01	.46E-02	-.43E-03	.12E-01					
-.11E-01	-.30E-01	.69E-02	.21E-01	.12E-01	.12E-01	.45	-1.0	.13E-01
.68	-.95E-02	.90E-01	-.16					
-.20E-02	.16E-01	.44E-04	-.65E-02	.44E-02	.20E-01	.12E-01	-.30E-01	.46E-02
-.95E-02	.38	.14E-01	.20E-01					
-.37E-02	.63E-02	-.20E-01	.14E-01	.18E-02	-.79E-02	.15	-.41	-.43E-03
.90E-01	.14E-01	.39	-.93E-02					
-.22E-01	-.40E-02	.82E-02	-.24E-02	.61E-02	.22E-01	-.95E-01	.28	.12E-01

-.16	.20E-01	-.93E-02	.44

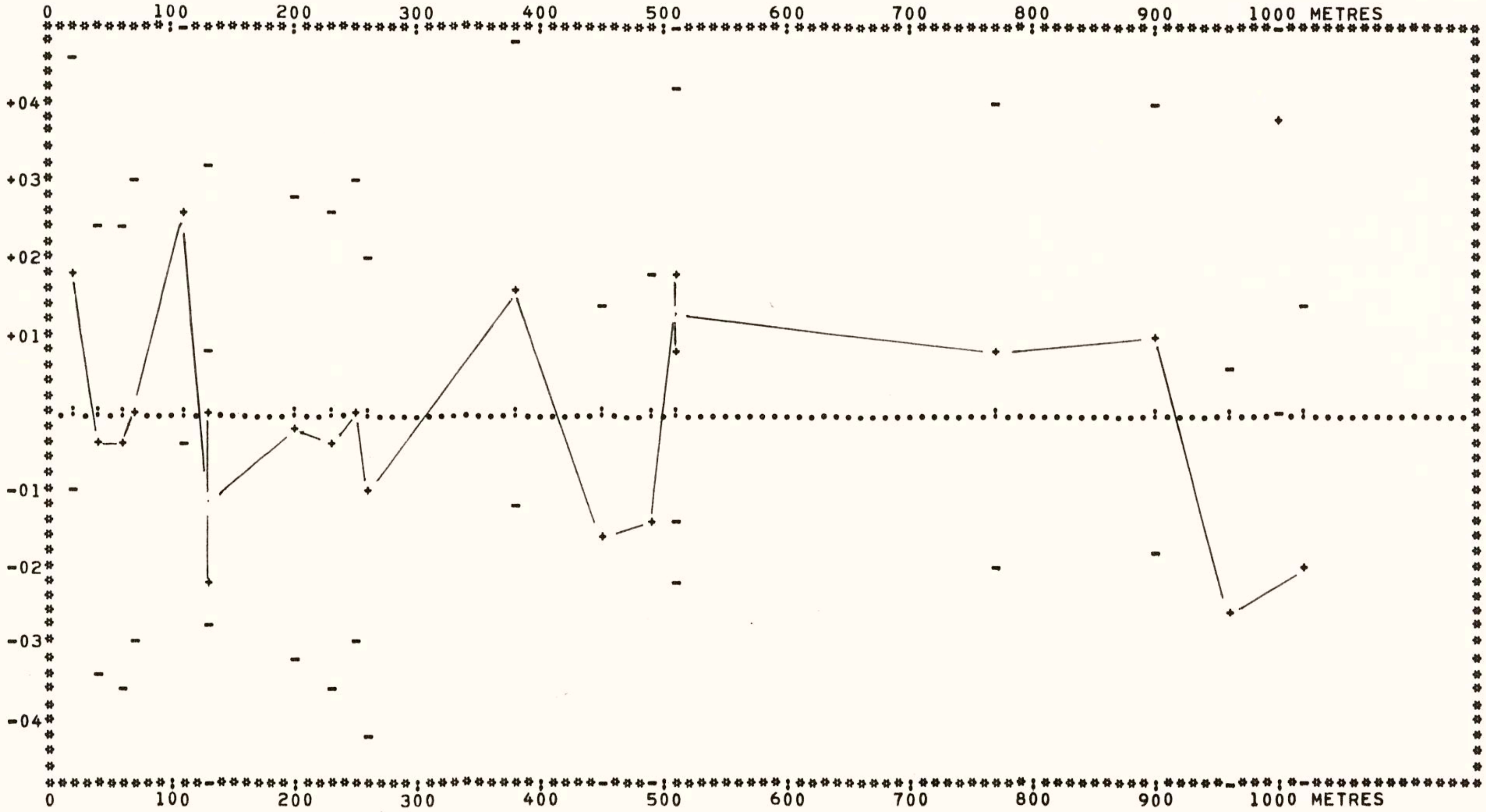
DIAGONAL ELEMENTS OF THIS MATRIX ARE :-

.21	.25	.30	.37	.46	.58	.78	3.9	.11
.68	.38	.39	.44					

MODE 1

RESIDUALS (+) WITH 95% CONFIDENCE INTERVALS (-) (USING A POSTERIORI VARIANCE FACTOR) VERSUS DISTANCE

SCALES HORIZONTAL 1:4000 (APPROX)
VERTICAL.. 16.3:1 (APPROX)



MODE 3

RESIDUALS (+) WITH 95% CONFIDENCE INTERVALS (-) (USING A POSTERIORI VARIANCE FACTOR) VERSUS DISTANCE

SCALES HORIZONTAL 1:4000 (APPROX)
VERTICAL.. 16.3:1 (APPROX)